

Warm Up:

Solve $2x^3 - 6x^2 + 2x - 6 = 0$
graphically.

X	Y ₁
-2	-50
-1	-16
0	-6
1	0
2	24

X=4



Real Root @ $x=3$

2 complex Roots



Solving Higher Degree Polynomials Algebraically

Remember the equation from the warm-up:

$$(2x^3 - 6x^2) + 2x - 6 = 0$$

$$2x^2(x-3) + 2(x-3) = 0$$

$$(x-3)(2x^2+2) = 0$$

$$(x-3)(2)(x^2+1) = 0$$

$$x-3=0$$

$$x=3$$

~~$$2=0$$~~

$$x^2+1=0$$

$$x^2 = -1$$

$$x = \pm i$$

$$\{3, i, -i\}$$

What is the degree of that equation?

3 (highest exp)

So how many solutions should it technically have?

3

How can this be?

one real
two imaginary

Solve the following algebraically (explain the different procedures):

4 terms:

Factor by Grouping

$$(x^3 - x^2) + (3x - 3) = 0$$

$$\underline{x^2}(x-1) + \underline{3}(x-1) = 0$$

$$(x^2 + 3)(x - 1) = 0$$

$$x^2 + 3 = 0 \quad x - 1 = 0$$

$$\begin{array}{r} -3 \quad -3 \\ \hline \sqrt{x^2 + 3} \end{array} \quad \boxed{x = 1}$$

$$x = \boxed{\pm \sqrt{3}i}$$

3 terms:

GCF, then quad.

$$x^3 + 2x^2 - 2x = 0$$

$$x(x^2 + 2x - 2) = 0$$

$$\boxed{x = 0} \quad | \quad x^2 + 2x - 2 = 0$$

$$x = \frac{-2 \pm \sqrt{4 + 8}}{2}$$

$$x = \frac{-2 \pm \sqrt{12}}{2} \quad \begin{array}{l} \sqrt{12} \\ \sqrt{4 \cdot 3} \\ 2\sqrt{3} \end{array}$$

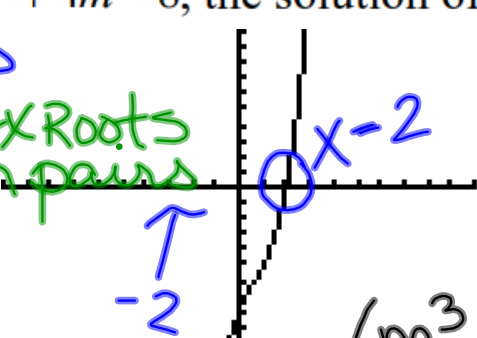
$$x = \frac{-2 \pm 2\sqrt{3}}{2}$$

$$x = \boxed{-1 \pm \sqrt{3}}$$

Given $c(m) = m^3 - 2m^2 + 4m - 8$, the solution of $c(m) = 0$ is

- ~~1) ± 2~~
- ~~2) 2, only~~
- 3) $2i, 2$
- 4) $\pm 2i, 2$

3 Roots
Complex roots come in pairs

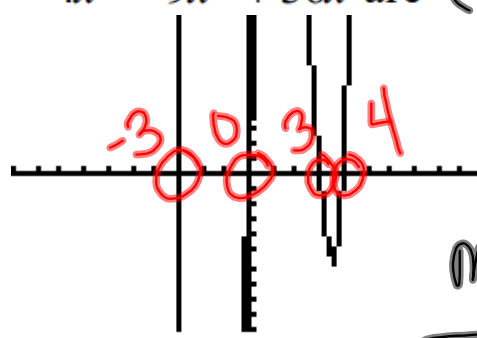


$$(m^3 - 2m^2) + (4m - 8)$$

$$m^2(m - 2) + 4(m - 2)$$

The zeros for $f(x) = x^4 - 4x^3 - 9x^2 + 36x$ are

- 1) $\{0, \pm 3, 4\}$
- 2) $\{0, 3, 4\}$
- ~~3) $\{0, \pm 3, -4\}$~~
- ~~4) $\{0, 3, -4\}$~~



$$(m - 2)(m^2 + 4)$$

$$m - 2 = 0 \quad m^2 + 4 = 0$$

$$m = 2 \quad \sqrt{m^2 + 4}$$

$$m = \pm 2i$$

**More
Practice:**

$$x^4 + 4x^3 + 4x^2 = -16x$$

$$8x^3 + 4x^2 - 18x - 9 = 0$$

$$x^3 - 3x^2 - 4x + 12 = 0$$

$$x^3 + 2x^2 - x - 2 = 0$$

Homework: p. 227 #1, 8, 9, 14, 20

Writing About Mathematics

1. Sharon said that if $f(x)$ is a polynomial function and $f(a) = 0$, then a is a root of the function. Do you agree with Sharon? Explain why or why not.

Developing Skills

In 3–18, find all roots of each given function by factoring or by using the quadratic formula.

8. $f(x) = -2x^3 + 6x^2 - x + 3$

9. $f(x) = x^4 - 5x^2 + 4$

14. $f(x) = x^5 - x^4 - 2x^3$

In 19–28: **a.** Find $f(a)$ for each given function. **b.** Is a a root of the function?

20. $f(x) = x^3 + 4x$, and $a = -2$