WARM UP

Solve the equation $3x^3 - 5x^2 - 48x + 80 = 0$ algebraically for all values of x.

$(3x^3 - 5x^2) + (-48x + 80) = 0$

$x^2(3x-5) - 16(3x-5) = 0$

$(3x-5)(x^2-16) = 0$

$(3x-5)(x+4)(x-4) = 0$

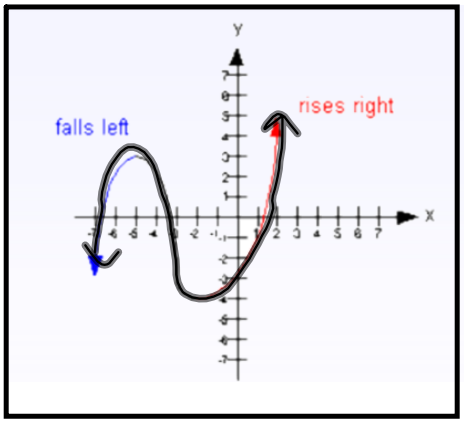
$3x-5=0 \quad x+4=0 \quad x-4=0$

$3x=5$

$x = \{\frac{5}{3}, -4, 4\}$

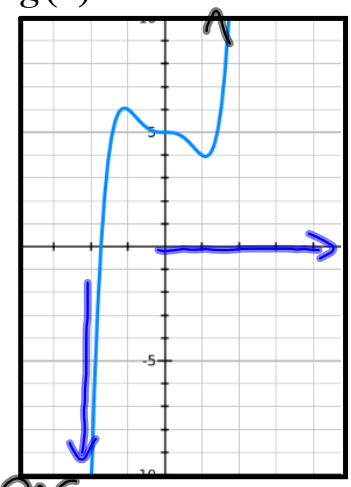
Relationships between Polynomials Equations and their Roots & Signs

Case 1: **POSITIVE ODD** (Meaning the leading coefficient is positive and it is an odd degree.)



Example:

$g(x) = x^5 - 2x^3 + 5$



Deg Odd: ends go in opposite directions

LC Pos: Right end goes up

End behavior:

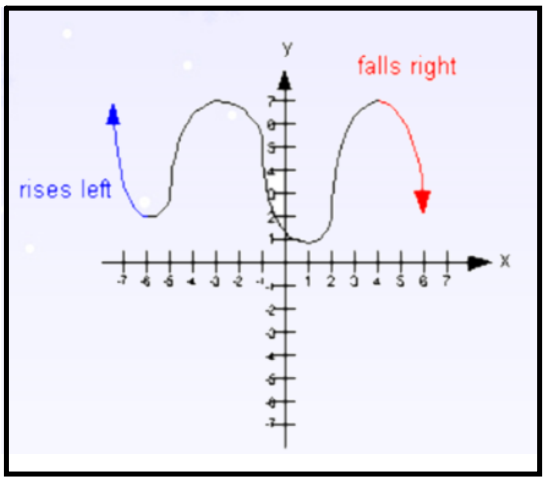
As $x \to \infty$, $y \to \infty$

$x \to -\infty$, $y \to -\infty$

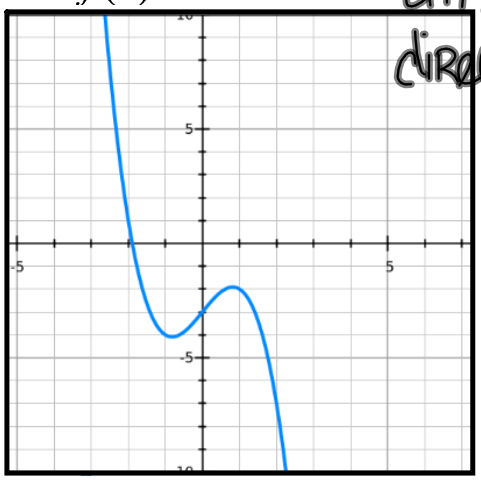
Case 2: NEGATIVE ODD (Meaning the leading coefficient is negative and it is an odd degree.)

Right side down

ends go in different directions



Example: $f(x) = -x^3 + 2x - 3$



End Behavior:

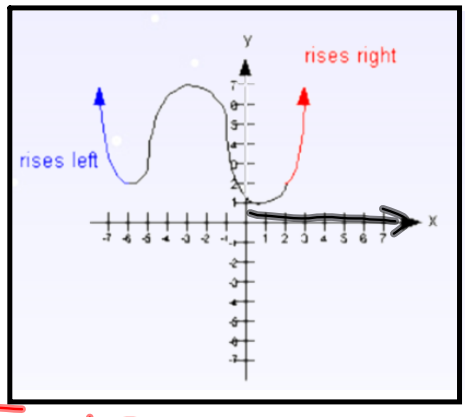
As $x \to \infty$, $y \to -\infty$

$x \to -\infty$, $y \to \infty$

Right side ↑

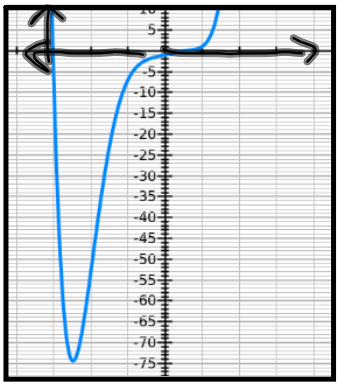
Case 3: POSITIVE EVEN (Meaning the leading coefficient is positive and it is an even degree.)

ends in same direction



Example:

$$h(x) = x^6 - 2x^5 - 3x^4 + 2x - 1$$

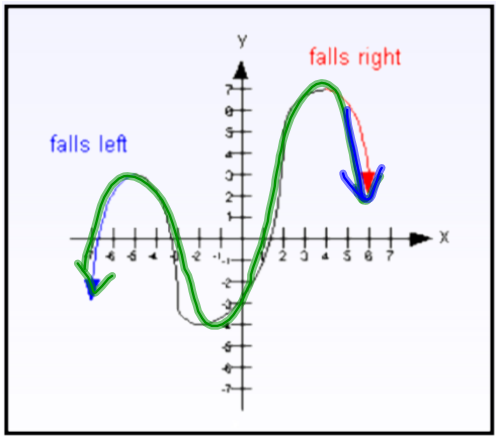


End Behavior:

$x \to \infty, y \to \infty$

$x \to -\infty, y \to \infty$

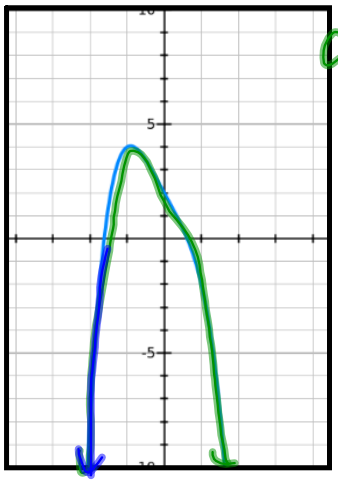
Case 4: NEGATIVE EVEN (Meaning the leading coefficient is negative and it is an even degree.)



Right ↓

Example.

$f(x) = -x^4 - 3x + 2$



ends Same direction

End Behavior:

$X \to \infty, y \to -\infty$

$X \to -\infty, y \to -\infty$

A ***zero*** or ***root*** of a polynomial function is the value of x such that $f(x) = 0$

Factor Theorem

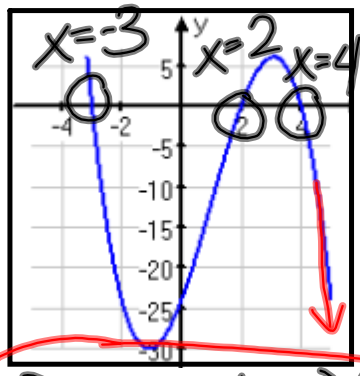
If $f(x)$ is a polynomial AND

1) $f(c) = 0$, then $x - c$ is a factor of $f(x)$.

2) $x - c$ is a factor of $f(x)$, then $f(c) = 0$.

$y = 0$

Roots are negations of the factors



x = -3 is a root
(x+3) is a factor

x = 2 is a root
(x-2) is a factor

x = 4 is a root
(x-4) is a factor

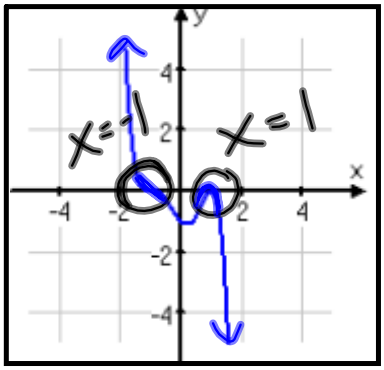
$f(x) = -(x+3)(x-2)(x-4)$

$(-x-3)(x^2-6x+8)$

$-x^3+6x^2-8x$

$-3x^2+18x-24$

$f(x) = -x^3+3x^2+10x-24$



x = -1 is a root
(flat???)
(x+1) is a factor

x = 1 is a root
(x-1) is a factor

$f(x) = -(x+1)(x-1)^2$

* If Roots look
- linear (once)
- parabola (twice)