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$$\begin{array}{r|rrrrr} -5 & 7 & 16 & 0 & -19 & -46 \\ & \downarrow & -35 & 95 & -475 & 2470 \\ \hline & 7 & -19 & 95 & -475 & \textcircled{2424} \end{array}$$

Warm Up:

Which equation does not have a solution?

Justify your answer.

a) $\sqrt{x-1}+3=4$

b) $\sqrt{x-2}+7=10$

c) $\sqrt{x+1}+3=4$

d) $\sqrt{x+2}-7=-10$

$3-4 \neq -10$
 $\sqrt{7+2}-7 = -10$

$\sqrt{x-1} = 1$

$x-1 = 1$

$x = 2 \checkmark$

$\sqrt{x-2} = 3$

$x-2 = 9$

$x = 11 \checkmark$

$\sqrt{x+1} = 1$

$x+1 = 1$

$x = 0 \checkmark$

$\sqrt{x+2} = -3$

$x+2 = 9$

$x = 7 \checkmark$

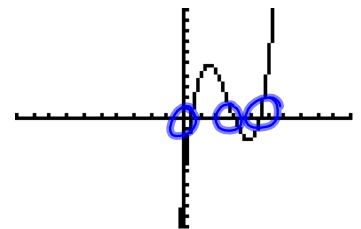


Finding ALL Roots

Find all the roots of the following equation & state the equation as a product of its linear factors.

$$f(x) = x^3 - 8x^2 + 17x - 6$$

Plot1 Plot2 Plot3
 $\sqrt{Y1} = X^3 - 8X^2 + 17X - 6$
 $\sqrt{Y2} =$
 $\sqrt{Y3} =$
 $\sqrt{Y4} =$
 $\sqrt{Y5} =$
 $\sqrt{Y6} =$



$x = 3$

1. Find a root graphically
2. Do synthetic (or long) division to find the next polynomial factor to find roots of
3. Repeat steps 1-2 until you're down to a quadratic
4. Solve the quadratic
5. Answer the question

$$\begin{array}{r|rrrr} 3 & 1 & -8 & 17 & -6 \\ & & 3 & -15 & 6 \\ \hline & 1 & -5 & 2 & 0 \end{array}$$

$\frac{1}{2}(-5) = -\frac{5}{2}$

$$f(x) = (x-3)(x^2 - 5x + 2)$$

$$x^2 - 5x + \frac{25}{4} = -2 + \frac{25}{4}$$

$$\left\{ 3, \left(\frac{5}{2} + \frac{\sqrt{17}}{2}\right), \left(\frac{5}{2} - \frac{\sqrt{17}}{2}\right) \right\}$$

$$\sqrt{\left(x - \frac{5}{2}\right)^2 - \frac{17}{4}}$$

$$x - \frac{5}{2} = \pm \frac{\sqrt{17}}{2}$$

$$x = \frac{5}{2} \pm \frac{\sqrt{17}}{2}$$

$$f(x) = (x-3)\left(x - \left(\frac{5}{2} + \frac{\sqrt{17}}{2}\right)\right)\left(x - \left(\frac{5}{2} - \frac{\sqrt{17}}{2}\right)\right)$$

Example:

a) Determine a possible rational root of $f(x) = 2x^3 + 11x^2 - 7x - 6$

a=2 b=13 c=6

$2x^2 + 13x + 6$

$(x-1) \overline{) 2x^3 + 11x^2 - 7x - 6}$

b) Solve for all roots

$x = \{1, -6, -\frac{1}{2}\}$

$x = \frac{-13 \pm \sqrt{169 - 48}}{4}$

$x = \frac{-13 \pm 11}{4} \rightarrow \frac{-13+11}{4} = -\frac{2}{4} = -\frac{1}{2}$

$\downarrow \frac{-13-11}{4} = -6$

$-\frac{6x-6}{6x-6}$

d) Write the polynomial as a product of linear factors

$f(x) = (x-1)(x+6)(2x+1)$

$(x^2 + 5x - 6)(2x+1)$

$2x^3$

Examples:

Find all roots of:

a) $f(x) = 2x^4 + x^3 + 5x + 4$

b) $f(x) = x^3 - 8x^2 + 17x - 6$

Find all the roots:

$$f(x) = 3x^4 - 10x^3 - 24x^2 - 6x + 5$$

Additional Practice:

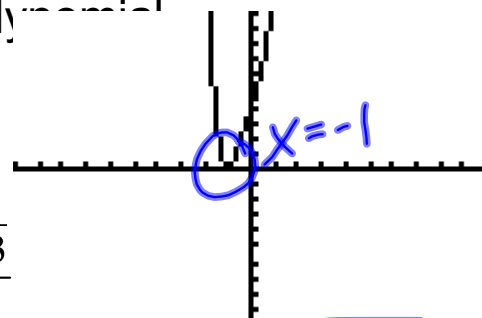
Find all Roots of the following Polynomials

1. $9x^3 + 6x^2 + 3x + 2 = 0$ $\left\{ -\frac{2}{3}, \pm i \frac{\sqrt{3}}{3} \right\}$

2. $2x^4 + x^3 + 5x + 4 = 0$ $\left\{ -1, -1, \frac{3 \pm i\sqrt{23}}{4} \right\}$

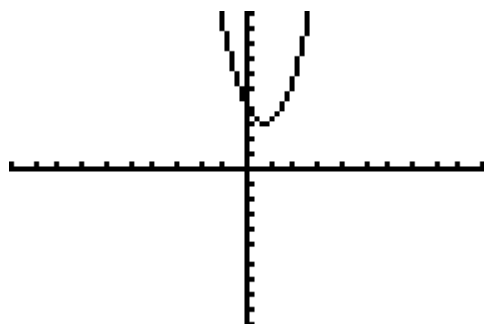
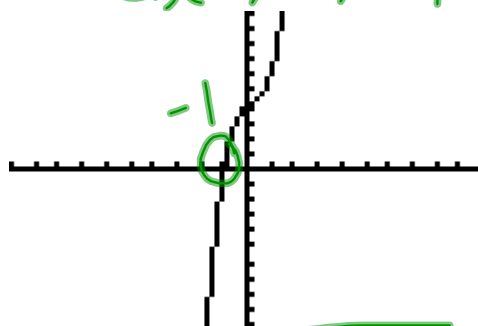
3. $3x^3 - 2x^2 - 2x + 8 = 0$ $\left\{ -\frac{4}{3}, 1 \pm i \right\}$

4. $x^3 - 4x^2 - 7x + 22 = 0$ $\{ 2, 1 \pm 2\sqrt{3} \}$



$$\begin{array}{r|rrrrr} -1 & 2 & 1 & 0 & 5 & 4 \\ \hline & \downarrow & -2 & 1 & -1 & -4 \\ \hline & & 2 & -1 & 4 & 0 \end{array}$$

$2x^3 - x^2 + x + 4 = 0$



$$\begin{array}{r|rrrr} -1 & 2 & -1 & 1 & 4 \\ \hline & \downarrow & -2 & 3 & -4 \\ \hline & & 2 & -3 & 4 & 0 \end{array}$$

$2x^2 - 3x + 4 = 0$