Answer Key

ALGEBRA 2 and TRIGONOMETRY

AMSCO

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Answers for Textbook Exercises

Chapter 1. The Integers

1-1 Whole Numbers, Integers, and the Number Line *(page 4)*

Writing About Mathematics

1. Answers will vary. Example: Have Tina count to three on her fingers, then count to two on her remaining fingers. Show her that if she counts the total number of fingers it equals five.
2. Yes. Both sides of the equation refer to the same distance along the number line.

Developing Skills

3. 6 **4.** 12 **5.** 5
6. 5 **7.** 7 **8.** 7
9. 4 **10.** 0 **11.** 0
12. 4 **13.** -5 **14.** 2
15. $8 + (-5) = 3$
16. $7 + (-(-2)) = 9$
17. $-2 + (-5) = -7$
18. $-8 + (-(-5)) = -3$
19. $\{-1, 11\}$

Applying Skills

20. $-\$20$
21. a. $-\$75$
b. $-\$23$
22. $+\$100$

1-2 Writing and Solving Number Sentences *(pages 8–9)*

Writing About Mathematics

1. Taking an absolute value always yields a positive number. There is no positive number that can be subtracted from 12 to yield 15.
2. No. Dividing both sides of an inequality by a negative number reverses the direction of the inequality.

Developing Skills

3. 7 **4.** 3 **5.** -2
6. -2 **7.** 4 **8.** -2
9. $\{9, -13\}$ **10.** $\{-5, 11\}$
11. $\{-1, 7\}$ **12.** $\{10, -13\}$
13. $a > 2, \{3, 4, 5, \ldots\}$ **14.** $b \geq 4, \{4, 5, 6, \ldots\}$
15. $1 < x < 3, \{2\}$ **16.** $3 < x < 7, \{4, 5, 6\}$
17. $-1 \geq b \geq 2, \{-1, 0, 1, 2\}$

Applying Skills

18. $156 - 3g \leq 9$, 49 cents
19. $5g + 3 = 18$, three groups
20. $12n + 8 = 80$, six nights
21. $5 + 3h = 44$, 13 hours
22. $19 + 5d < 49$, five plants
23. $5d + 4 \leq 14$, two hours

1-3 Adding Polynomials *(pages 12–13)*

Writing About Mathematics

1. Yes. If x is negative, $|2x + 1|$ will always be greater than x. If x is positive (or zero), $2x + 1$ is always greater than x.
2. No. Terms in each polynomial may or may not have like terms in the other polynomial. Furthermore, if like terms have coefficients with equal value but opposite signs, adding them will eliminate a term with that power. Thus, the sum of a trinomial and a binomial may have anywhere from zero to five terms.

Developing Skills

3. $5y - 13$ **4.** $5x^2 + x + 1$
5. $7x^2 - 5x - 4$ **6.** $3x$
7. $2a^2b^2 + 2$ **8.** $4b^2 - 10b$
9. $-6 - 3b$ **10.** $x^2 + 7x - 8$
11. $4y^2 - 3y - 4$ **12.** $2a^4 + a^3 - 5a^2 + a - 1$
13. 6 **14.** 4
15. -5 **16.** -3
17. $x < 12, \{\ldots, 9, 10, 11\}$
18. $y \geq 2, \{2, 3, 4, \ldots\}$
19. $y \leq -1, \{\ldots, -3, -2, -1\}$
20. $c > 4, \{5, 6, 7, \ldots\}$
21. 1
22. $x \leq -5, \{\ldots, -7, -6, -5\}$

Applying Skills

23. \$2.00
24. a. $6x + 10$
b. 4 feet wide and 13 feet long
25. 50 cents

1-4 Solving Absolute Value Equations and Inequalities *(pages 16–17)*

Writing About Mathematics

1. The absolute value of a number is equal to the absolute value of its negative.

2. Subtract 7 from both sides. The absolute value is then equal to a negative number, which makes the solution set empty.

Developing Skills

3. $\{-7, 17\}$
4. $\{-2, -14\}$
5. $\{-1, 6\}$
6. $\{-3, 7\}$
7. $\{1, 7\}$
8. $\{3, -4\}$
9. $\{5, 9\}$
10. $\{3, -3\}$
11. $\{2, -10\}$
12. $\{-3, 8\}$
13. $\varnothing$
14. $\{-3, 17\}$
15. $x < -9$ or $x > 9$, $\{\ldots, -12, -11, -10, 10, 11, 12, \ldots\}$
16. $x < -9$ or $x > 5$, $\{\ldots, -12, -11, -10, 6, 7, 8, \ldots\}$
17. $-11 \leq b \leq -1$, $\{-11, -10, -9, \ldots, -3, -2, -1\}$
18. $-1 < y < 7$, $\{0, 1, 2, 3, 4, 5, 6\}$
19. $y < -19$ or $y > 7$, $\{\ldots, -22, -21, -20, 8, 9, 10, \ldots\}$
20. $b \leq -1$ or $b \geq 8$, $\{\ldots, -3, -2, -1, 8, 9, 10, \ldots\}$
21. $-3 < x < 7$, $\{-2, -1, 0, 1, 2, 3, 4, 5, 6\}$
22. The set of integers
23. $0 < b < 10$, $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$
24. $b < -3$ or $b > 14$, $\{\ldots, -6, -5, -4, 15, 16, 17, \ldots\}$
25. $\varnothing$
26. $-3 \leq b \leq 17$, $\{-3, -2, -1, \ldots, 15, 16, 17\}$
27. $\{253, 254, 255, 256, 257, 258, 259\}$, $253 \leq x \leq 259$
28. $\{150, 151, 152, 153, \ldots, 297, 298, 299, 300\}$, $150 \leq t \leq 300$
29. $|c - 200| \leq 28$, solution $= 172 \leq c \leq 228$, $\{172, 173, 174, \ldots, 226, 227, 228\}$

1-5 Multiplying Polynomials *(page 21)*

Writing About Mathematics

1. No. Using FOIL, the answer is $(a + 3)(a + 3) = a^2 + 6a + 9$.
2. Six. Each term of the trinomial (3) is multiplied by each term of the binomial (2).

Developing Skills

3. $14a^8b^4$
4. $-12c^3d^4$
5. $36x^2y^4$
6. $9c^8$
7. $-9c^8$
8. $15b^2 - 12b$
9. $2x^2y^2 - 4x^2y^3$
10. $2x^2 + 5x - 3$
11. $a^2 - a - 20$
12. $3x^2 - 5x - 2$
13. $a^2 - 9$
14. $25b^2 - 4$
15. $a^2 + 6a + 9$
16. $9b^2 - 12b + 4$
17. $y^3 - 3y^2 + 3y - 1$
18. $2x^3 + 5x^2 - 7x - 15$
19. $11a - 12$
20. $4b^2 + 5b$
21. $8y^2 - 7y - 10$
22. 0
23. $z^3 - 6z^2 + 12z - 8$
24. -5
25. 3
26. 2
27. 1
28. 6
29. 4

Applying Skills

30. $2x^2 + 4x$
31. **a.** $a = c - 1, b = c - 8$
    b. $a^2 + b^2 = (c - 1)^2 + (c - 8)^2$
    $= 2c^2 - 18c + 65$
    c. $2c^2 - 18c + 65$

1-6 Factoring Polynomials *(pages 26–27)*

Writing About Mathematics

1. Yes. If we multiply these factors back together, we get $x^2 + (d + e)x + de$.
2. No. These factors will yield +4 as the last term instead of -4.

Developing Skills

3. $4x(2x + 3)$
4. $3a^2(2a^2 - a + 3)$
5. $5ab(b - 3 + 4a)$
6. $x^2y^2(xy - 2x + 1)$
7. $4a(1 - 3b + 4a)$
8. $7(3a^2 - 2a + 1)$
9. $(y - 1)(y + 1)$
10. $(3b - 4)(b - 2)$
11. $(2x + 3)(y + 4)$
12. $(a^2 + 3)(a - 3)$
13. $(x^2 - 2)(2x - 3)$
14. $(y^2 - 5)(y + 1)$
15. $(x + 7)(x + 1)$
16. $(x + 3)(x + 2)$
17. $(x - 3)(x - 2)$
18. $(x + 6)(x - 1)$
19. $(x - 3)(x + 2)$
20. $(x + 4)(x + 5)$
21. $(3x + 4)(x - 3)$
22. $(2y - 1)(y + 3)$
23. $(5b + 1)(b + 1)$
24. $(6x - 1)(x - 2)$
25. $(2y + 1)(2y + 1)$
26. $(3x - 2)(3x - 2)$
27. $(a + 3)(a + 1)(a - 1)$
28. $5(x - 1)(x - 2)$
29. $b(b + 2)(b - 2)$
30. $4a(x + 3)(x - 2)$
31. $3(2c + 1)(2c - 1)$
32. $(x^2 + 9)(x + 3)(x - 3)$
33. $(x^2 + 4)(x + 2)(x - 2)$
34. $x(2x + 3)(x + 5)$
35. $2x(2x - 3)(x - 1)$
36. $(z^2 - 3)(z + 3)(z - 3)$
37. $(c + 3)(c + 1)$
38. $(y + 1)(-y + 3) = -1(y - 3)(y + 1)$
39. $y(x + 4)(x - 4)$
40. $3(x - 3)(x + 1)$
41. $-9(x + 1)(x + 3)$

Applying Skills

42. $(4x + 1)(x - 2)$
43. $(4x + 5)(4x - 5)$
44. $(3x - 1)(3x - 1)$
45. $(3x - 1)(x + 2)$

1-7 Quadratic Equations with Integral Roots *(pages 29–30)*

Writing About Mathematics

1. No. If the product of two expressions is zero, at least one of the two expressions must be zero. This is not true for other numbers.

2. Yes. If the product of any number of expressions is zero, at least one of the expressions must be zero.

Developing Skills

3. {1, 3}
4. {2, 5}
5. {−1, 6}
6. {−1, −5}
7. {2, −12}
8. {−1, 10}
9. {−1, 4}
10. {3, −10}
11. {−2, 3}
12. {−3, 4}
13. {1, 7}
14. {3}
15. {−3, 2}
16. {1}
17. {5}

Applying Skills

18. Francis is 11, Brad is 14.
19. Length: 30 ft, width: 18 ft
20. Width: 8 ft, length: 18 ft
21. 9 cm, 12 cm, 15 cm
22. 3 seconds

1-8 Quadratic Inequalities *(page 35)*

Writing About Mathematics

1. No. If all three factors are negative, the product will be negative. Furthermore, if two factors are negative, the product will be positive.
2. **a.** Yes. The solution set is $5 < x < 7$; thus, any value makes $(x - 7)$ the negative factor and $(x - 5)$ the positive factor.
   b. No. We can tell for binomial factors of the form $(x + a)(x + b)$ where a and b are given. However, in other products, such as xy, either factor can be the positive factor.

Developing Skills

3. $-3 < x < -2, \varnothing$
4. $x < -6$ or $x > 1$, {. . . , −9, −8, −7, 2, 3, 4, . . .}
5. $1 \leq x \leq 2$, {1, 2}
6. $x < 2$ or $x > 5$, {. . . , −1, 0, 1, 6, 7, 8, . . .}
7. $-2 < x < 3$, {−1, 0, 1, 2}
8. $x \leq -2$ or $x \geq 10$, {. . . , −4, −3, −2, 10, 11, 12, . . .}
9. $-4 < x < 3$, {−3, −2, −1, 0, 1, 2}
10. $x < 1$ or $x > 5$, {. . . , −2, −1, 0, 6, 7, 8, . . .}
11. $x \leq 0$ or $x \geq 2$, {. . . , −2, −1, 0, 2, 3, 4, . . .}
12. $-2 < x < 3$, {−1, 0, 1, 2}
13. $x < 2$ or $x > 2$, {. . . , −1, 0, 1, 3, 4, 5, . . .}
14. The set of integers
15. $-2 < x < 1$, {−1, 0}
16. $-3 \leq x \leq 4$, {−3, −2, −1, 0, 1, 2, 3, 4}
17. $x < -3$ or $x > 4$, {. . . , −6, −5, −4, 5, 6, 7, . . .}

Applying Skills

18. {1 ft by 2 ft, 2 ft by 3 ft, 3 ft by 4 ft, 4 ft by 5 ft, 5 ft by 6 ft, 6 ft by 7 ft}
19. {1 ft by 3 ft by 3 ft, 2 ft by 4 ft by 3 ft, 3 ft by 5 ft by 3 ft, 4 ft by 6 ft by 3 ft}

Review Exercises *(pages 37–38)*

1. $-2x$
2. $-a + 12$
3. $-3d + 7$
4. $3b^2 - 25b + 45$
5. $x^2 + 7x - 20$
6. $-2a^2 - 2a$
7. $14d^2 + 19cd - 3c^2$
8. $x^2 - x - 1$
9. 4
10. 0
11. $2x^2$
12. $y^2 - 4y$
13. $2(x + 1)(x + 3)$
14. $3(a - 5)(a - 5)$ or $3(a - 5)^2$
15. $5x(x + 1)(x - 4)$
16. $10a(b + 2)(b - 2)$
17. $(c^2 + 4)(c + 2)(c - 2)$
18. $3(y^2 + 2)(y - 4)$
19. $(x - 1)(x + 1)(x + 5)$
20. $(x + 1)(x - 1)(x + 1)(x - 1)$
21. $2(x - 3)(x - 6)$
22. $x(x - 2)(x - 1)$
23. $5(a^2 + b^2)(a + b)(a - b)$
24. $(5x - 3)(x + 5)$
25. −9
26. 13
27. $x > 4$, {5, 6, 7, . . .}
28. $-1 \leq x < 4$, {−1, 0, 1, 2, 3}
29. {2, −7}
30. {6, −8}
31. $y < -1$ or $y > 2$, {. . . , −4, −3, −2, 3, 4, 5, . . .}
32. $x < -5$ or $x > -1$, {. . . , −8, −7, −6, 0, 1, 2, . . .}
33. {4, 5}
34. {5, 7}
35. $-6 < x < -1$, {−5, −4, −3, −2}
36. $x < -5$ or $x > 7$, {. . . , −8, −7, −6, 8, 9, 10, . . .}
37. $0 \leq x \leq 5$, {0, 1, 2, 3, 4, 5}
38. $x < -3$ or $x > 0$, {. . . , −6, −5, −4, 1, 2, 3, . . .}
39. $1 \leq x \leq 3$, {1, 2, 3}
40. $x \leq -2$ or $x \geq 1$, {. . . , −4, −3, −2, 1, 2, 3, . . .}
41. An absolute value cannot be equal to a negative number.
42. Width: 12 cm, length: 32 cm
43. Width: 8 ft, length: 30 ft
44. 10 in., 24 in., 26 in.
45. **a.** 96 feet
    b. 1 second and 4 seconds

Exploration *(page 38)*

1. 6; 28; 496; 8,128
2. All Euclidean perfect numbers have 2^{k-1} as a factor. Since k is always a positive integer greater than 1, Euclidean perfect numbers will be multiples of 2.

3. The possible units digits of any power of 2 are $\{2, 4, 6, 8\}$.
Given the units digit of any 2^{k-1}, the units digit of $(2^k - 1) = 2(2^{k-1}) - 1$.
If $2^{k-1} = 2$, then the units digit of $(2^k - 1) = 3$.
If $2^{k-1} = 4$, then the units digit of $(2^k - 1) = 7$.
If $2^{k-1} = 6$, then the units digit of $(2^k - 1) = 1$.
If $2^{k-1} = 8$, then the units digit of $(2^k - 1) = 5$.
However, integers with units digit 5 (other than 5 itself) are not prime, so $(2^k - 1) \neq 5$ and $2^{k-1} \neq 8$.
The product of 2×3 and of 6×1 is 6. The product of 4×7 ends in 8. Therefore, a Euclidean perfect number $N = 2^{k-1}(2^k - 1)$ must have a units digit of 6 or 8.

Chapter 2. The Rational Numbers

2-1 Rational Numbers *(page 43)*

Writing About Mathematics

1. a. The coin is called a quarter because it is 25 out of 100 cents, one-fourth the value of a dollar.
   b. A quarter of something is equivalent to one-fourth of its total value. Since the total number of minutes in an hour and cents in a dollar differ, one-fourth of those values will also differ.
2. The additive inverse makes the sum of the two numbers equal zero. The multiplicative inverse makes the product of the two numbers equal to one.

Developing Skills

3. $\frac{8}{3}$ 4. $\frac{12}{7}$ 5. $\frac{-7}{2}$
6. $\frac{1}{8}$ 7. 1
8. $0.166\ldots = 0.1\overline{6}$
9. $0.222\ldots = 0.\overline{2}$
10. $0.7142857142\ldots = 0.\overline{714285}$
11. $0.133\ldots = 0.1\overline{3}$
12. $0.875\overline{0}$
13. $\frac{1}{8}$ 14. $\frac{2}{3}$ 15. $\frac{2}{9}$
16. $\frac{4}{11}$ 17. $\frac{4}{37}$ 18. $\frac{47}{300}$
19. $\frac{5}{6}$ 20. $\frac{26}{45}$ 21. $\frac{3}{22}$
22. $\frac{7}{44}$

2-2 Simplifying Rational Expressions *(pages 47–48)*

Writing About Mathematics

1. Abby is wrong. $3x$ is not a common factor of the numerator and denominator, and cannot be canceled out.
2. No. It is true for all values *except* where the denominator is zero $\left(a = \frac{3}{2}\right)$.

Developing Skills

3. $a = 0$ 4. $c = 0$
5. $a = 0, b = 0$ 6. $x = -5$
7. $a = \frac{7}{2}$ 8. $b = 2, -3$
9. $c = 0, 1$ 10. $x = -1, 0, 6$
11. $\frac{3}{5}$ 12. $\frac{ab}{2}$ $(a \neq 0)$
13. $\frac{4y}{x}$ $(x \neq 0, y \neq 0)$ 14. $\frac{2b}{3}$ $(b \neq 0)$
15. $\frac{3}{4c^3}$ $(c \neq 0, d \neq 0)$ 16. $\frac{2a + 4}{3a}$ $(a \neq 0)$
17. $\frac{3y + 1}{2y}$ $(y \neq 0)$ 18. $\frac{4a - 2b}{3a}$ $(a \neq 0, b \neq 0)$
19. $\frac{2}{3 - 4d}$ $\left(d \neq 0, \frac{3}{4}\right)$ 20. $\frac{c}{c + 2}$ $(c \neq 0, -2)$
21. $\frac{1}{3 + 2xy^2}$ $\left(x \neq 0, y \neq 0, xy^2 \neq -\frac{3}{2}\right)$
22. $\frac{2}{3}$ $(a \neq -5)$ 23. $a - 2$ $(a \neq -2)$
24. $\frac{x - 4}{x + 5}$ $(x \neq 3, -5)$ 25. $\frac{5(y - 2)}{y + 2}$ $(y \neq -2)$
26. $\frac{1}{3a + 3}$ $(a \neq -1, 1)$ 27. $a + 1$ $(a \neq 1)$
28. $\frac{1}{b + 2}$ $(b \neq 2, -2)$ 29. $\frac{-2}{x - 3}$ $(x \neq 3)$
30. $-\frac{5}{b + 4}$ $(b \neq -4, 4)$

2-3 Multiplying and Dividing Rational Expressions *(pages 52–53)*

Writing About Mathematics

1. No. Joshua needed to write the reciprocal of the second fraction before attempting to cancel out any common factors.
2. Yes. He divided the terms separately, which is acceptable based upon the commutative property of multiplication.

Developing Skills

3. $\frac{1}{2}$ 4. $\frac{3}{28}$ $(a \neq 0)$
5. $\frac{1}{10}$ $(x \neq 0, y \neq 0)$ 6. $\frac{2}{3}$ $(a \neq 0)$
7. $\frac{3}{5}$ $(b \neq -1)$ 8. $\frac{a^2 + 10a}{6}$ $(a \neq 0, 10)$
9. $\frac{3}{y^2 - 3y}$ $(y \neq -3, 0, 3)$
10. $\frac{2(a - 1)}{3(a + 4)}$ $(a \neq -4, -2, 4)$
11. 1 $(a \neq 0, -2)$ 12. $\frac{-5}{2x}$ $(x \neq -3, 0, 3)$
13. $\frac{5}{3}$ 14. 8 $(a \neq 0)$
15. 4 $(b \neq 0, c \neq 0)$ 16. $\frac{1}{6}$ $(a \neq 0)$

17. $\frac{3}{4x}$ $(x \neq 0, 2)$

18. $\frac{2}{2y + 1}$ $\left(y \neq -\frac{1}{2}, 0, \frac{1}{2}\right)$

19. 1 $(c \neq 3)$

20. $\frac{1}{w + 1}$ $(w \neq -1, 0, 1)$

21. $\frac{4}{b}$ $(b \neq 0, -3)$

22. $\frac{a + 5}{4a}$ $(a \neq 0, -3)$

23. $(2x + 7)^2(x - 1)$ $\left(x \neq 1, \frac{-7}{2}\right)$

24. $\frac{a^2 - a}{2}$ $(a \neq -1)$

25. $\frac{1}{4}$

26. 9 $(x \neq -1, 0, 1)$

27. $\frac{1}{2}$ $(a \neq -2, 0, 2)$

28. $\frac{(x - 1)(x + 4)}{x}$ $(x \neq 0, 1)$

29. $6(b + 2)^2$ $(b \neq 0)$

30. $3x$ $(x \neq 0, 1, 2)$

2-4 Adding and Subtracting Rational Expressions *(pages 56–57)*

Writing About Mathematics

1. No. It is also undefined when $a = 1$.
2. Yes. He formed a correct LCD and added.

Developing Skills

3. x

4. $\frac{-5x^2 + 2}{5x}$ $(x \neq 0)$

5. $\frac{10x}{21}$

6. $\frac{-a - 9}{20}$ or $-\frac{a + 9}{20}$

7. $\frac{7y}{6}$

8. $\frac{3a + 80}{40a}$ $(a \neq 0)$

9. $\frac{x + 12}{12x}$ $(x \neq 0)$

10. $\frac{11a + 2}{6a}$ $(a \neq 0)$

11. $\frac{3x + 2}{x}$ $(x \neq 0)$

12. $\frac{10y - 1}{2y}$ $(y \neq 0)$

13. $\frac{2a^2 - 3}{2a}$ $(a \neq 0)$

14. $\frac{1 + x}{x}$ $(x \neq 0)$

15. $\frac{x^2 + 3x - 4}{x(x + 2)}$ $(x \neq 0, -2)$

16. $\frac{2b + 1}{2(b - 1)}$ $(b \neq 1)$

17. $\frac{1}{x - 2}$ $(x \neq 2)$

18. $\frac{1}{(2a - 1)(a + 2)}$ $\left(a \neq -2, -\frac{1}{2}, -3\right)$

19. $\frac{1}{a(a - 2)}$ $(a \neq -2, 0, 2)$

20. $\frac{2}{x}$ $(x \neq 0, 2)$

Applying Skills

21. **a.** $\frac{4x^2 + 2}{x}$ **b.** 2

22. **a.** $\frac{18x + 20}{3}$ **b.** $x + 1$

23. **a.** $\frac{2x + 6}{x - 1}$ **b.** $\frac{3x}{(x - 1)(x - 1)}$

24. **a.** $\frac{4x^2 + 6x}{(x + 1)(x + 2)}$ **b.** $\frac{x^2}{(x + 1)(x + 1)}$

2-5 Ratio and Proportion *(pages 60–61)*

Writing About Mathematics

1. Yes. Interchanging the means or extremes of a proportion maintains the equality of the proportion.
2. Yes. One is added to each side of the equation, which maintains the equality.

Developing Skills

3. 3 : 2 **4.** 3 : 2 **5.** 1 : 6
6. 1 : 5 **7.** 2 : 3 **8.** 2 : 3
9. 1 : 3 **10.** 2 : 7 **11.** 2
12. 7 **13.** 7 **14.** $\frac{1}{3}$
15. 7 **16.** $\{-3, 2\}$ **17.** $\{0, 5\}$
18. $\{-1, 4\}$ **19.** $\{-10, 2\}$

Applying Skills

20. 16 inches wide by 20 inches long
21. 28 inches long by 12 inches wide
22. 15 games
23. 33 members
24. \$75 and \$50
25. $2\frac{1}{4}$ cups
26. 4 cups of solution and 28 cups of water

2-6 Complex Rational Expressions *(pages 63–64)*

Writing About Mathematics

1. $\{-1, 0, 1\}$. An expression of the form $\frac{1}{x} \div \frac{y}{z} = \frac{1}{x} \cdot \frac{z}{y}$ is undefined when either x, y, or z is zero.
2. No. When $\frac{d^2}{2} = 2$, the denominator will equal zero, which would make the fraction undefined.

Developing Skills

3. 4

4. $\frac{1}{10}$

5. $\frac{1}{2}$

6. 4 $(x \neq 0)$

7. $\frac{1}{2}$

8. $\frac{1}{a}$ $(a \neq -1, 0)$

9. -2 $(d \neq 0, 1)$

10. $-(b + 1)$ $(b \neq 0, 1)$

11. $\frac{3}{b}$ $(b \neq 0, 1)$

12. $\frac{1}{2}$ $\left(y \neq -\frac{1}{2}\right)$

13. $\frac{2(2y + 1)}{y}$ $\left(y \neq 0, \frac{1}{2}\right)$

14. $\frac{5x}{6}$ $(x \neq 0)$

15. $\frac{(a + 7)}{(a - 2)}$ $(a \neq 0, 2, 7)$

16. $\frac{3}{x - 5}$ $(x \neq 0, 3, 5)$

17. $\frac{b + 2}{b - 1}$ $(b \neq -1, 0, 1)$

18. $\frac{y - 2}{y + 8}$ $(y \neq -8, -3, 0)$

19. -1 $(a \neq 0, 1)$

20. $-\frac{5(a + 3)}{a}$ $(a \neq 0, 3)$

21. $\frac{1}{2x}$ $(x \neq 0, -1)$

22. $\frac{17}{4a}$ $(a \neq 0)$

23. $\frac{3a + 2}{4a}$ $\left(a \neq 0, \frac{5}{3}\right)$

24. -1 $(b \neq -2, 0, 2)$

2-7 Solving Rational Equations *(pages 69–70)*

Writing About Mathematics

1. Yes. Samantha multiplied both sides by the LCD, which is a valid way to solve this equation.
2. Brianna is correct. A rational equation has a variable in one or more denominators.

Developing Skills

3. 32	**4.** 8	**5.** 8
6. 70	**7.** 12	**8.** 20
9. 80	**10.** 20	**11.** 8
12. 10	**13.** 12	**14.** $-\frac{8}{5}$
15. $\frac{1}{2}$	**16.** $\left\{-2, \frac{3}{2}\right\}$	**17.** 3
18. $\{5, -7\}$	**19.** 4	**20.** $\frac{7}{3}$

Applying Skills

21. Week 1: Joseph worked 8 hours and Nicole worked 12.
Week 2: Joseph worked 12 hours and Nicole worked 24.

22. 5 mph

23. 40 mph then 50 mph

24. Price: $1.25, 6.6 lb the 1st week, 7.6 lb the 2nd week

2-8 Solving Rational Inequalities *(pages 73–74)*

Writing About Mathematics

1. The number line must be separated by the solutions to the equation as well as the values at which any of the rational expressions are undefined.

2. $\{x : x < 0\}$. Since the numerator will be positive for any nonzero rational number and undefined at 0, the expression is negative for all $x < 0$.

Developing Skills

3. $a < -24$	**4.** $y < 8$
5. $b > \frac{2}{5}$	**6.** $d < 2$
7. $a > \frac{153}{5}$	**8.** $0 < x < 1$
9. $0 < y < 4$	**10.** $a < -2$ or $a > -1$
11. $\frac{5}{3} < x < 4$	**12.** $x < 0$ or $x > \frac{1}{2}$
13. $-7 < x < -5$	**14.** $-5 < a < -1$

Review Exercises *(pages 75–76)*

1. $\frac{2}{7}$

2. $\{-1, 0, 1\}$

3. $0.41\overline{6}$

4. $\frac{2a}{3b}$ $(a \neq 0, b \neq 0)$

5. $\frac{13}{20a}$ $(a \neq 0)$

6. $\frac{3}{2x(x - 4)}$ $(x \neq -4, 0, 4)$

7. $\frac{a^2}{5a + 25}$ $(a \neq -5, -2, 0)$

8. $\frac{2a + 1}{a^2 - 1}$ $(a \neq -1, 1)$

9. $\frac{y + 6}{y + 3}$ $(y \neq -3, 3)$

10. $\frac{d - 3}{d}$ $(d \neq -6, 0)$

11. $\frac{1}{2}$ $(b \neq 0)$

12. $\frac{2a + 2}{a^2 + 2a}$ $(a \neq -2, -1, 0)$

13. $a + 4$ $(a \neq 4)$

14. $\frac{x - 1}{x - 2}$ $(x \neq -2, -1, 2)$

15. $-\frac{x + 1}{x}$ $(x \neq 0, 1)$

16. $\frac{2}{3}$

17. $\frac{a}{b}$ $(a \neq 0, b \neq 0, a \neq -b)$

18. $\frac{1}{x + 6}$ $(x \neq -6, 6)$

19. $\frac{a(a - 5)}{a - 4}$ $(a \neq -4, 0, 4)$

20. $\frac{1}{11}$

21. $\frac{20}{3}$

22. 6

23. $\{0, 8\}$

24. 2

25. $\left\{-2, \frac{3}{2}\right\}$

26. $x < 0$ or $x > 2$

27. $x < -\frac{1}{2}$ or $x > -\frac{9}{20}$

28. 27 boys, 30 girls

29. Week 1: 15 cans, week 2: 12 cans, $0.70 per can

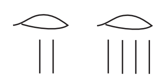
30. 60 mph then 45 mph

31. 16 ft by 13 ft and 14 ft by 13 ft

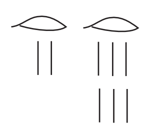
Exploration *(page 76)*

1.
$$\frac{1}{n + 1} + \frac{1}{n(n + 1)} = \frac{n + 1}{n(n + 1)} = \frac{1}{n}$$

2.



$\frac{3}{4} = \frac{1}{2} + \frac{1}{4}$



or

$\frac{2}{3} = \frac{1}{2} + \frac{1}{6}$ $\qquad$ $\frac{2}{3} = \frac{1}{3} + \frac{1}{4} + \frac{1}{12}$

3. **a.** $\frac{1}{3} + \frac{1}{15}$ $\qquad$ **b.** $\frac{1}{2} + \frac{1}{5}$

c. $\frac{1}{2} + \frac{1}{12}$ $\qquad$ **d.** $\frac{1}{2} + \frac{1}{3} + \frac{1}{8}$

e. $\frac{1}{2} + \frac{1}{9}$

Cumulative Review *(pages 77–78)*

Part I

1. 4	**2.** 1	**3.** 3
4. 2	**5.** 1	**6.** 1
7. 2	**8.** 2	**9.** 3
10. 1		

Part II

11. *Answer:* $2 \leq x \leq 5$

$$|7 - 2x| \leq 3$$
$$7 - 2x \leq 3 \text{ and } 7 - 2x \geq -3$$
$$x \geq 2 \qquad\qquad x \leq 5$$

12. $\frac{3}{a+5} - \frac{a-3}{5} \div \frac{a^2-9}{15}$

$= \frac{3}{a+5} - \frac{a-3}{5} \cdot \frac{15}{(a-3)(a+3)}$

$= \frac{3}{a+5} - \frac{3}{a+3}$

$= \frac{3(a+3) - 3(a+5)}{(a+5)(a+3)}$

$= \frac{-6}{(a+5)(a+3)}\ (a \neq -5, -3, 3)$

Part III

13. $\frac{5x+5}{x^2-1} \cdot \frac{x^2-x}{15}$

$= \frac{5(x+1)}{(x-1)(x+1)} \cdot \frac{x(x-1)}{15}$

$= \frac{x}{3}\ (x \neq -1, 1)$

14. Width: 6 m, length: 15 m

$$l = \frac{w}{2} + 12$$

$$w\left(\frac{w}{2} + 12\right) = 90$$

$$\frac{w^2}{2} + 12w - 90 = 0$$

$$w^2 + 24w - 180 = 0$$

$$(w + 30)(w - 6) = 0$$

$w = -30, w = 6$. Reject negative value.

Part IV

15. *Answer:* $x < -1$ or $x > \frac{7}{2}$

$$2x^2 - 5x > 7$$

$$2x^2 - 5x - 7 > 0$$

$$(2x - 7)(x + 1) > 0$$

The solutions for the corresponding equations are $\frac{7}{2}$ and -1.

When $x < -1$, the inequality is true.

When $-1 < x < \frac{7}{2}$, the inequality is false.

When $x > \frac{7}{2}$, the inequality is true.

16. Diego traveled at 60 mph and then at 20 mph.

$$\frac{30}{x} + \frac{10}{\frac{x}{3}} = 1$$

$$\frac{30}{x} + \frac{30}{x} = 1$$

$$\frac{60}{x} = 1$$

$$x = 60$$

Chapter 3. Real Numbers and Radicals

3-1 The Real Numbers and Absolute Value *(page 83)*

Writing About Mathematics

1. No. The expression can be written as the ratio $\left(\frac{15}{4}\right)$ of two integers, so it is rational.
2. No. Maria's inequality is a false statement. If she applied the rule "$|x| > k$, then $x > k$ or $x < -k$," she would get $-2x + 5 > 3$ or $2x - 5 < -3$.

Developing Skills

3. rational	**4.** irrational
5. irrational	**6.** irrational
7. rational	**8.** irrational
9. irrational	**10.** rational
11. rational	**12.** irrational
13. rational	**14.** irrational

In 15–26, answers will be graphs of number lines.

15. $-7 < x < 7$	**16.** $a \geq 8$ or $a \leq 2$
17. $y > 2$ or $y < -7$	**18.** $-1 \leq b \leq 2$
19. $a < -9$ or $a > -1$	**20.** $-4 < x < 2$
21. $x > \frac{2}{5}$ or $x < -\frac{1}{5}$	**22.** { } or $\varnothing$
23. all real numbers	**24.** $x = \frac{-4}{5}$
25. all real numbers	**26.** all real numbers

Applying Skills

27. $70° \leq t \leq 220°$

28. 282 ft $\leq h \leq$ 20,320 ft

3-2 Roots and Radicals *(pages 87–88)*

Writing About Mathematics

1. **a.** Yes. Since the product of an even number of either positive or negative factors is positive, the radical will have both a positive and a negative root.
   b. Yes. The product of an odd number of positive factors is positive, and the product of an odd number of negative factors is negative. Thus a radical with a real, nonzero radicand and an odd index will have only one real root.
2. **a.** Yes. This is true only when $a \geq 0$. The other equal factor is $-\sqrt{a}$.
   b. The statement is true for $a \geq 0$. When $a < 0$, a has no square roots in the set of real numbers.

Developing Skills

3. rational	**4.** irrational
5. neither	**6.** rational
7. rational	**8.** rational
9. rational	**10.** rational

11. 4	**12.** ± 4	**13.** -4
14. 25	**15.** 13	**16.** -0.2
17. ± 0.8	**18.** 1.2	**19.** 3
20. 2	**21.** -5	**22.** -5
23. 5	**24.** -1	**25.** $\frac{2}{5}$
26. $-\frac{7}{6}$	**27.** $\frac{2}{3}$	**28.** $-\frac{1}{2}$
29. 0.1	**30.** 0.4	**31.** x^3

32. $10c^2$ **33.** $0.5x$ **34.** $\frac{10}{a}$
35. $-\frac{b^2}{6}$ **36.** $-0.1y$ **37.** 1
38. x^2 **39.** $x \geq 2$ **40.** $x \leq 3$
41. $x \geq -3$ **42.** $x \geq -5$ **43.** $x = \pm 9$
44. $a = \pm 14$ **45.** $b = \pm 10$ **46.** $y = \pm 13$

Applying Skills

47. $\sqrt{14}$ cm $\approx$ 3.74 cm
48. $\sqrt{544}$ ft $\approx$ 23.32 ft
49. 15 in.
50. 6 in.
51. 13 ft

3-3 Simplifying Radicals *(pages 93–94)*

Writing About Mathematics

1. $-\sqrt{36}$ is the negative of the square root of 36, which is a real number, simplifying to -6. $\sqrt{-36}$ is the square root of a negative number and is not real.
2. Negative. If a is negative, $-8a^3$ will be positive and its cube root will be also positive. The negative sign in front makes the whole expression negative.

Developing Skills

3. $2\sqrt{3}$ **4.** $5\sqrt{2}$
5. $4\sqrt{2}$ **6.** $2b\sqrt{2b}$
7. $7c^2\sqrt{2}$ **8.** $6y^2\sqrt{5y}$
9. $50y\sqrt{2x}$ **10.** $44x^2y^3\sqrt{3xy}$
11. $3b^2\sqrt{2ab}$ **12.** $2\sqrt[3]{2}$
13. $2\sqrt[3]{3}$ **14.** $2a\sqrt[3]{5a}$
15. $5xy^2\sqrt[3]{3x^2}$ **16.** $2a^2\sqrt[4]{3ab^3}$
17. $\frac{2a^2}{5}$ **18.** $\frac{b}{7}\sqrt{3b}$
19. $\frac{\sqrt{6}}{9y^3}$ **20.** $\frac{a^2}{2}\sqrt{2a}$
21. $\frac{a}{5b}\sqrt{5ab}$ **22.** $\frac{\sqrt{6xy}}{6xy}$
23. $\frac{\sqrt{3xy}}{2y}$ **24.** $\frac{\sqrt{15b}}{5b^3}$
25. $\frac{\sqrt{10a}}{6}$ **26.** $\frac{\sqrt{30b}}{4b^2}$
27. $\frac{2a}{9b^2}\sqrt{3ab}$ **28.** $\frac{3}{10xy^4}\sqrt{2xy}$
29. $\frac{2}{5y^2}\sqrt{10xy}$ **30.** $\frac{\sqrt{2}}{5}$
31. $\frac{\sqrt{2}}{2}$ **32.** $\frac{11b^2\sqrt{2ab}}{10}$
33. $\frac{x}{100}\sqrt{10x}$ **34.** $8x\sqrt{2}$
35. $10\sqrt{3c}$ **36.** $\frac{a}{3}\sqrt[3]{9}$
37. $\frac{a^2\sqrt[4]{2b^2c}}{bc}$ **38.** $2xy\sqrt[4]{2x}$

Applying Skills

39. $4\sqrt{13}$ cm **40.** $6\sqrt{2}$ in.
41. $12\sqrt{3}$ m **42.** $5\sqrt{13}$ ft
43. $5\sqrt{6}$ ft **44.** $xy^2\sqrt{5}$ m
45. $6y^2$ units
46. The longest diagonal of the trunk is $\sqrt{1{,}604}$ or approximately 40.05 inches. Thus, everything but the walking stick will fit.

3-4 Adding and Subtracting Radicals *(pages 97–98)*

Writing about Mathematics

1. Yes, for $x > 0$. $(3x)^2 = 9x^2$, and $\sqrt{9x^2} = 3x$. Her substitution is correct.
2. No. We do not add radicands. $\sqrt{16} + \sqrt{48} = 4 + 4\sqrt{3}$, which is not equal to $\sqrt{64} = 8$.

Developing Skills

3. $6\sqrt{2}$ **4.** $2\sqrt{5}$
5. $9\sqrt{3}$ **6.** $4\sqrt{7}$
7. $6\sqrt{2}$ **8.** $\sqrt{5y}$
9. $6a\sqrt{10}$ **10.** $5b^2\sqrt{11}$
11. $5y\sqrt{6x}$ **12.** $5a^3\sqrt{2a}$
13. $4c^2\sqrt{2c}$ **14.** $12x\sqrt{2x}$
15. $11b^2\sqrt{6b}$ **16.** $22x^3\sqrt{5}$
17. $\frac{6\sqrt{5}}{5}$ **18.** $3\sqrt{6}$
19. $4\sqrt{7}$ **20.** $\frac{\sqrt{2x}}{x}$
21. $5a\sqrt{5} - 5\sqrt{2a}$ **22.** $22x\sqrt{6}$
23. $6\sqrt{3y} + y^2$ **24.** $6a^2b\sqrt{2b} + 2$
25. $9\sqrt{3} - 2\sqrt{6}$ **26.** $3\sqrt{5} - \frac{\sqrt{10}}{10}$
27. $\frac{1}{2}\sqrt{6}$ **28.** $3\sqrt[3]{2}$
29. $7\sqrt[3]{2}$ **30.** $\sqrt[4]{3}$
31. $8\sqrt{x}$ **32.** $5\sqrt{y}$
33. $\sqrt{2a}$ **34.** $23b\sqrt{2}$
35. $3a\sqrt{7} - 3a\sqrt{5}$ or $3a(\sqrt{7} - \sqrt{5})$
36. $b\sqrt{a}$ **37.** $15x\sqrt{2x}$
38. $3x\sqrt{x}$ **39.** $\sqrt{3}$
40. $\frac{\sqrt{2}}{2}$ **41.** $\sqrt{6}$
42. $\frac{\sqrt{5}}{3}$

Applying Skills

43. $14\sqrt{3}$ in.

44. $25\sqrt{3}$ ft

45. a. $5\sqrt{2}$ cm

b. $12\sqrt{2}$ cm

46. a. 14 in.

b. $14\sqrt{2} + 14$ in. or $14(1 + \sqrt{2})$ in.

47. a. $34\sqrt{10}$ m

b. $13\sqrt{10}$ m

3-5 Multiplying Radicals *(pages 100–101)*

Writing About Mathematics

1. Yes. $\sqrt{2}$ is a positive real number.
2. Yes. We can simplify by dividing the exponent of the radicand by the index.

Developing Skills

3. 4 **4.** 15

5. 9 **6.** $4\sqrt{6}$

7. $-6\sqrt{5}$ **8.** $6\sqrt{5}$

9. $2\sqrt{2}$ **10.** $2\sqrt{7}$

11. $4\sqrt{10}$ **12.** 12

13. 27 **14.** 20

15. $2x^2$ **16.** $4ab\sqrt{b}$

17. $2y^2\sqrt{5}$ **18.** $x^3y^2\sqrt{3}$

19. $\frac{35a}{3}$ **20.** $\frac{x}{2}\sqrt{x}$

21. $\frac{a}{6}\sqrt{3a}$ **22.** 2

23. $3a^2\sqrt[3]{5}$ **24.** 3

25. $2\sqrt{2} + 2$ **26.** $\sqrt{5} - 5\sqrt{2}$

27. $12\sqrt{2} + 4$ **28.** $5a - 3\sqrt{5a}$

29. $6xy^2 + 6y\sqrt{3xy}$ **30.** $-2 + 2\sqrt{5}$

31. $9 + 10\sqrt{2b} + 2b$ **32.** $21 - 4\sqrt{5y} - 5y$

33. $49 - 5b$ **34.** $2x^2 - 3x\sqrt[4]{3y} + \sqrt{3y}$

35. $-36 - \sqrt{6}$ **36.** $6 - 36c^2$

37. $a^2 - b$ **38.** $4 - 2\sqrt{3}$

39. $9 + 6b\sqrt{5ab} + 5ab^3$ **40.** $-6 - 6\sqrt{7}$

41. $-2 - \sqrt{5}$

Applying Skills

42. 4,608 m^2 **43.** 120 ft^2

44. $\sqrt{5}$ $in.^3$

45. a. $2\sqrt{6}$ ft **b.** $6 + 2\sqrt{6}$ ft **c.** 3 ft^2

46. $\pi(4 + xy^5 + 4y^2\sqrt{xy})$ m^2

3-6 Dividing Radicals *(pages 103–104)*

Writing About Mathematics

1. No. Jonathan's error was treating the denominator of $\frac{\sqrt{10}}{2}$ as $\sqrt{2}$. $\frac{\sqrt{10}}{2}$ does not simply further.
2. Answers may vary. Example: $\frac{\sqrt{27}}{\sqrt{3}} = \sqrt{9} = 3$, which is rational. $\frac{\sqrt{6}}{\sqrt{3}} = \sqrt{2}$, which is irrational.

Developing Skills

3. 2 **4.** 5

5. 3 **6.** $a\sqrt{10}$

7. $\frac{2\sqrt{2x}}{x}$ **8.** $5\sqrt{2}$

9. $3\sqrt{3}$ **10.** $2\sqrt{3}$

11. $\frac{a\sqrt{14}}{2}$ **12.** $\frac{2\sqrt{6xy}}{3y}$

13. $\frac{3\sqrt{2b}}{2b}$ **14.** $\frac{\sqrt{3x}}{x}$

15. $\frac{\sqrt{7y}}{y}$ **16.** $\sqrt{3a}$

17. $c\sqrt{2}$ **18.** $2 + 4\sqrt{6}$

19. $\sqrt{2} - 3\sqrt{10}$ **20.** $2 + \sqrt{3}$

21. 1 **22.** 5

23. $1 + \frac{1}{2}\sqrt{6}$ **24.** $\sqrt{5} + 6$

25. $\sqrt[3]{9} + \sqrt[3]{12x^2}$ **26.** $\frac{1}{a}\sqrt[4]{a^3}$ or $\frac{\sqrt[4]{a^3}}{a}$

27. $\sqrt{c}$ **28.** $\frac{2\sqrt[3]{w}}{w}$

29. $8\sqrt{x} + \sqrt[4]{40}$

Applying Skills

30. $\frac{5}{2}\sqrt{7}$ ft

31. a. $2\sqrt{6}$ cm

b. 6 cm

3-7 Rationalizing a Denominator *(pages 107–108)*

Writing About Mathematics

1. **a.** If Juan writes 7 as $\sqrt{49}$, the fraction becomes $\frac{\sqrt{49}}{2\sqrt{7}}$. This is equivalent to $\frac{1}{2}\sqrt{\frac{49}{7}}$, which simplifies to $\frac{1}{2}\sqrt{7}$.

   b. No. Juan's procedure cannot be applied to $\frac{7}{2\sqrt{5}}$ because 5 is not a factor of 49.
2. Brittany took the fraction at face value and multiplied by the conjugate of the denominator. Justin saw that the denominator factored to $2(1 + \sqrt{2})$. 2 is a factor of the numerator, so the fraction is equivalent to $\frac{2}{1 + \sqrt{2}}$.

Developing Skills

3. $\frac{\sqrt{3}}{3}$
4. $\frac{1}{2}\sqrt{10}$
5. $2\sqrt{2}$
6. $\frac{2}{3}\sqrt{3}$
7. $\sqrt{3}$
8. $\frac{\sqrt{6}}{12}$
9. $\frac{4}{3}\sqrt{3}$
10. $\sqrt{3}$
11. $\frac{2}{3}\sqrt{6}$
12. $\frac{1}{6}\sqrt{6}$
13. $\frac{1}{6}\sqrt{10}$
14. 1
15. $\frac{3 - \sqrt{5}}{4}$
16. $\frac{5 + \sqrt{2}}{23}$
17. $\frac{\sqrt{3} - 1}{2}$
18. $\frac{6 + 2\sqrt{3}}{3}$
19. $\frac{3\sqrt{5} - 3}{4}$
20. $\frac{16 - 4\sqrt{7}}{9}$
21. $3\sqrt{5} + 6$
22. $3\sqrt{7} - 6$
23. $\sqrt{2} + 1$
24. $3 - \sqrt{3}$
25. $\frac{5x + 2\sqrt{5x}}{5x - 4}$
26. $\frac{10y\sqrt{y} - 2\sqrt{5y}}{5y^2 - 1}$
27. $\frac{8 + 5\sqrt{2}}{7}$
28. $\frac{17 + 2\sqrt{7}}{9}$
29. $\frac{11 - 2\sqrt{10}}{9}$
30. $\frac{27 + 7\sqrt{5}}{22}$
31. $\frac{(a + 2)(b + \sqrt{2})}{b^2 - 2}$
32. $\frac{2x + 5y - 7\sqrt{xy}}{x - y}$
33. $\frac{4\sqrt{z} - 32}{z - 64}$
34. $\frac{a + 2\sqrt{a}}{a - 4}$
35. $\frac{2y\sqrt{x} + 2x\sqrt{y}}{xy}$
36. $\frac{3\sqrt{x} + (36 - x)\sqrt{6} - 18}{3x - 108}$
37. $\frac{2a\sqrt{ab} + 2b\sqrt{ab}}{ab}$ or $\frac{2\sqrt{ab}}{ab}(a + b)$
38. $\frac{3x^2 - 3x\sqrt{2} + (5x^2 - 10)\sqrt{x}}{x(x^2 - 2)}$
39. **a.** $\frac{2}{3}\sqrt{6}$
    b. 1.632993162
    c. 1.632993162
40. **a.** $\frac{2\sqrt{3} + 3}{3}$ or $\frac{2}{3}\sqrt{3} + 1$
    b. 2.154700538
    c. 2.154700538
41. **a.** $2\sqrt{3} + 2$
    b. 5.464101615
    c. 5.464101615
42. **a.** $8 + 3\sqrt{7}$
    b. 15.93725393
    c. 15.93725393
43. $\sqrt{2}$
44. $\sqrt{3}$
45. $\sqrt{3} - 1$
46. $\sqrt{8} - \sqrt{5}$

Applying Skills

47. $(6\sqrt{5} - 6)$ in.
48. $\frac{3}{2}\sqrt{2}$ ft, $\frac{3}{2}\sqrt{2}$ ft, $2\sqrt{2}$ ft

3-8 Solving Radical Equations
(pages 112–113)

Writing About Mathematics

1. There is no real number for which the square root is negative. If $x \geq -3$ the radicand will *not* be negative, so there will be a solution in the set of real numbers.
2. No. Once we square the equation it has two real roots. One is the root of the given equation and the other is the root of $\sqrt{15 - 7x} = 1 - x$.

Developing Skills

3. 25	**4.** 49	**5.** 9
6. 36	**7.** 16	**8.** 32
9. 4	**10.** 8	**11.** 4
12. 44	**13.** 4	**14.** 2
15. 2	**16.** 25	**17.** 4
18. 4	**19.** 5	**20.** 2
21. −1	**22.** 5	**23.** 5, 4
24. { } (no solution)		
25. 5	**26.** 3	**27.** 1
28. 3	**29.** $\left\{1, \frac{2}{3}\right\}$	**30.** 8
31. −10	**32.** 22	**33.** −34
34. 18	**35.** 4	**36.** {0, 8}
37. 5	**38.** 5	

Applying Skills

39. 8 units each
40. Width = 2, length = 1
41. **a.** $AB = BC = 2\sqrt{10}$, $AC = \sqrt{10}$
    b. $5\sqrt{10}$

Review Exercises *(pages 114–116)*

In 1–8, answers will be graphs of number lines.

1. $-\frac{1}{4} < x < \frac{3}{4}$
2. $-6.5 \leq x \leq 2.5$
3. $-10.5 \leq x \leq 11.5$
4. $x < -2$ or $x > 4.5$
5. $x \leq \frac{-3}{5}$ or $x \geq 2\frac{1}{5}$
6. $x > 1$ or $x < -13$
7. $-1 \leq x \leq 2\frac{1}{5}$
8. $0 < x < 2$
9. $8\sqrt{2}$
10. $\frac{\sqrt{3}}{2}$
11. $15\sqrt{3}$
12. $\sqrt{5}$
13. $21\sqrt{3}$
14. $4\sqrt{2}$
15. $5\sqrt{6} + 2\sqrt{5}$
16. 12
17. 75
18. $4 + 3\sqrt{2}$

19. $20 - 20\sqrt{2}$ **20.** $6\sqrt{2} + 60$

21. -1 **22.** 22

23. $5 + 3\sqrt{3}$ **24.** $\sqrt{2}$

25. $2\sqrt{5} + 1$ **26.** $4\sqrt{2} - 2$

27. $\frac{4\sqrt{3} + 3}{13}$ **28.** $8\sqrt{a}$

29. $7b\sqrt{2b}$ **30.** $x\sqrt[4]{12x}$

31. $\frac{\sqrt{ab}}{b}$ **32.** $12x^2\sqrt{x}$

33. $4x - x\sqrt[4]{36}$ **34.** $x^2y^2\sqrt{y}$

35. $3a^3\sqrt[3]{2}$ **36.** $4a^2$

37. $b^2\sqrt[3]{9}$ **38.** $\frac{3 + \sqrt{x}}{9 - x}$

39. $\frac{16 + a - 8\sqrt{a}}{16 - a}$ **40.** a

41. x^2 **42.** 13

43. 5 **44.** 18

45. 14 **46.** $\{3, 4\}$

47. 5 **48.** -5

49. $\{-2, 3\}$ **50.** $12 + 7\sqrt{3}$ ft

51. **a.** 4 m
b. $4 + 4\sqrt{2}$ m

52. Show that these values satisfy the Pythagorean Theorem:

$$x^2 + \left(\sqrt{2x + 1}\right)^2 = (x + 1)^2$$
$$x^2 + 2x + 1 = x^2 + 2x + 1$$

True for all values of $x > 0$ because of the radical.

53. **a.** $24 - 12\sqrt{3}$ ft
b. $52 - 22\sqrt{3}$ ft

Exploration *(page 116)*

Answers will vary.

Cumulative Review *(pages 116–118)*

Part I

1. 4 **2.** 3 **3.** 4
4. 3 **5.** 1 **6.** 3
7. 4 **8.** 4 **9.** 4
10. 1

Part II

11.
$$\frac{a^2 - 16}{a^2 - a - 12} \div \frac{a^2 + 4a}{2a}$$
$$= \frac{a^2 - 16}{a^2 - a - 12} \cdot \frac{2a}{a^2 + 4a}$$
$$= \frac{(a + 4)(a - 4)}{(a + 3)(a - 4)} \cdot \frac{2a}{a(a + 4)}$$
$$= \frac{2}{a + 3}$$

12.
$$2x^2 - 9x = 5$$
$$2x^2 - 9x - 5 = 0$$
$$(2x + 1)(x - 5) = 0$$

$x = -\frac{1}{2}$

Check: $x = -\frac{1}{2}$

$$2\left(-\tfrac{1}{2}\right)^2 - 9\left(-\tfrac{1}{2}\right) \stackrel{?}{=} 5$$
$$2\left(\tfrac{1}{4}\right) + \tfrac{9}{2} \stackrel{?}{=} 5$$
$$5 = 5 \checkmark$$

$x = 5$

Check: $x = 5$

$$2(5)^2 - 9(5) \stackrel{?}{=} 5$$
$$2(25) - 45 \stackrel{?}{=} 5$$
$$5 = 5 \checkmark$$

Part III

13.
$$|3b + 6| < 7$$
$$-7 < 3b + 6 < 7$$
$$-13 < 3b < 1$$
$$\frac{-13}{3} < b < \frac{1}{3}$$
$$-4\tfrac{1}{3} < b < \tfrac{1}{3}$$

14. No. correct = 2(no. incorrect) + 4
Let no. incorrect = x
No. correct = $2x + 4$

$$\frac{2x + 4}{x} = \frac{8}{3}$$
$$8x = 6x + 12$$
$$2x = 12$$
$$x = 6$$

Plugging back in, Tyler answered 16 questions correctly.

Part IV

15. Use $D = RT$, where D is distance, R is speed, and T is time. Then $T = \frac{D}{R}$.
Let x = Rachel's speed on local streets, and $2x$ be her highway speed, both in mph.

$$\frac{2}{x} + \frac{12}{2x} = \frac{16}{60}$$
$$\frac{2 + 6}{x} = \frac{16}{60}$$
$$\frac{8}{x} = \frac{16}{60}$$
$$16x = 480$$
$$x = 30$$

Rachel travels 30 mph on local streets and 60 mph on the highway.

16. let w = the width in yards. The length is therefore $l = 4w + 2$.
Use $A = lw$.

$$w(4w + 2) = 30$$
$$4w^2 + 2w - 30 = 0$$
$$2w^2 + w - 15 = 0$$
$$(2w - 5)(w + 3) = 0$$
$$w = 2\tfrac{1}{2} \quad\Big|\quad w = -3$$

Reject negative value.

The garden is $2\frac{1}{2}$ yards wide and 12 yards long.

Chapter 4. Relations and Functions

4-1 Relations and Functions
(pages 126–127)

Writing About Mathematics

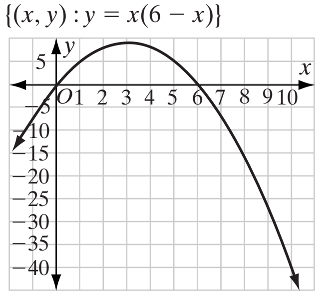
1. $\{(x, y) : x = y^2\}$ is not a function because for all values of $x > 0$ there are two distinct y-values, whereas $\{(x, y{:}\sqrt{x} = y\}$ is a function because for every value of $x \geq 0$ there is exactly one real number that is the square root. No two pairs have the same first element.
2. No, because not every positive integer has an integral square root. The range contains non-integer values.

Developing Skills

3. **a.** function; no two pairs have the same first element
   b. $\{1, 2, 3, 4\}$
   c. $\{1, 4, 9, 16\}$
4. **a.** not a function; points $(1, -1)$ and $(1, 1)$ have the same first element
   b. $\{0, 1\}$
   c. $\{-1, 0, 1\}$
5. **a.** function; no two pairs have the same first element
   b. $\{-2, -1, 0, 1, 2\}$
   c. $\{5\}$
6. **a.** function
   b. {all real numbers}
   c. $\{y : y \geq -1\}$
7. **a.** not a function
   b. {all real numbers}
   c. $\{y : y \leq -1 \text{ or } y \geq 1\}$
8. **a.** not a function
   b. $\{x : x \geq -2\}$
   c. {all real numbers}
9. **a.** function
   b. {all real numbers}
   c. range: $y = 2$
10. **a.** function
    b. $\{x : -3 \leq x \leq 3\}$
    c. $\{y : 0 \leq y \leq 4\}$
11. **a.** function
    b. $\{x : 1 \leq x \leq 6\}$
    c. $\{y : 0 \leq y \leq 2.5\}$
12. **a.** {all real numbers}
    b. The function is not onto since the range is $\{-183\}$.
13. **a.** {all real numbers}
    b. The function is onto since the range is equal to the domain.
14. **a.** {all real numbers}
    b. The function is not onto since the range is $\{y : y \geq 0\}$.
15. **a.** {all real numbers}
    b. The function is not onto since the range is $\left\{y : y \leq \frac{1}{4}\right\}$.
16. **a.** $\{x : x \geq 0\}$
    b. The function is onto since the range is equal to the domain.
17. **a.** {all real numbers}
    b. The function is not onto since the range is $\{y : y \geq 0\}$.
18. **a.** $\{x : x \neq 0\}$
    b. The function is onto since the range is equal to the domain.
19. **a.** $\{x : x \leq 3\}$
    b. The function is onto since the range is equal to the domain.
20. **a.** $\{x : x > -1\}$
    b. The function is not onto since the range is $\{y : y > 0\}$.
21. **a.** {all real numbers}
    b. The function is not onto since the range is $\{y : 0 < y \leq 1\}$.
22. **a.** $\{x : x \neq 1\}$
    b. The function is onto since the range is equal to the domain.
23. **a.** $\{x : x \neq 3\}$
    b. The function is not onto since the range is $\{y : y < 1\}$.

Applying Skills

24. **a.** $\{(x, y) : y = x(6 - x)\}$
    b.



x	0	1	2	3	4	5	6	7	8	9	10
y	0	5	8	9	8	5	0	−7	−16	−27	−40

    c. $\{x : 0 < x < 6\}$
25. **a.** $\{(x, y) : y = 10x\}$
    b. (0, 0), (1, 10), (2, 20), (3, 30), (4, 40), (5, 50), (6, 60), (7, 70), (8, 80)
    c. $\{0, 1, 2, 3, 4, 5, 6, 7, 8\}$
    d. $\{0, 10, 20, 30, 40, 50, 60, 70, 80\}$

4-2 Function Notation *(pages 128–129)*

Writing About Mathematics

1. f and g are the same function, since evaluating g at x yields the same value as f.
2. f and g are not the same function. For example, note that $f(3) = 9$, while $g(3) = g(1 + 2) = 3$.

Developing Skills

3. a. $f(x) = x - 2$
b. 3

4. a. $f(x) = x^2$
b. 25

5. a. $f(x) = |3x - 7|$
b. 8

6. a. $f(x) = 5x$
b. 25

7. a. $f(x) = \sqrt{x - 1}$
b. 2

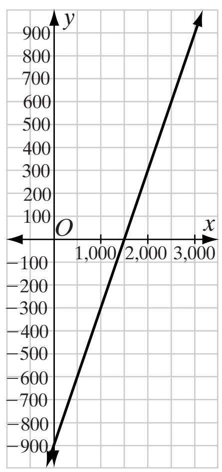
8. a. $f(x) = \frac{2}{x}$
b. $\frac{2}{5}$

9. 10 **10.** 10 **11.** 12
12. 4 **13.** 2 **14.** 1
15. a. 2 **b.** −3 **c.** −2 **d.** 2

Applying Skills

16. a. $t(a) = 0.08a$
b. $\{a : a \geq 0\}$
c. $0.40
d. $1.32

17. a.



b. $300
c. 3,000 muffins

4-3 Linear Functions and Direct Variation *(pages 133–135)*

Writing About Mathematics

1. Yes. $g(x) = \frac{1}{a}f(x)$, when $a > 1$, is equivalent to $f(x) = ag(x)$, and $ag(x)$ is the graph of $g(x)$ stretched vertically by a factor of a.
2. Yes. Directly proportional means the ratio of $r : s$ is constant. Every direct variation of two variables is a linear function that is one-to-one.

Developing Skills

3. a. $\{1, 2, 3, 4\}$
b. $\{4, 7, 10, 13\}$
c. yes

4. a. $\{0, 2, 4, 6\}$
b. $\{8, 6, 4, 2\}$
c. yes

5. a. $\{2, 3, 4, 5, 6\}$
b. $\{7\}$
c. no

6. a. $\{0, -1, -2, -3, -4\}$
b. $\{3, 5, 7, 9, 11\}$
c. yes

7. no **8.** no **9.** no
10. yes **11.** no **12.** no

13. a. graph
b. yes
c. yes

14. a. graph
b. no
c. yes

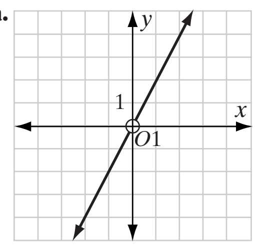
15. a. graph
b. yes
c. yes

16. a. graph
b. no
c. yes

17. a. graph
b. yes
c. yes

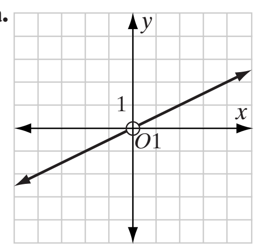
18. a. graph
b. no
c. yes

19. a.



b. yes
c. yes

20. a.



b. yes
c. yes

21. no
22. yes

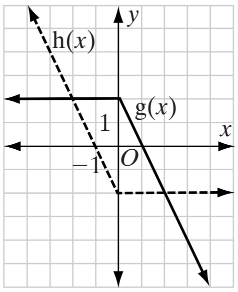
Applying Skills

23. $\frac{c}{n} = 6$ **24.** $\frac{d}{t} = 35$ **25.** $\frac{i}{f} = 12$
26. $\frac{g}{k} = 1{,}000$ **27.** $\frac{g}{m} = 25$

28. a. $g(t) = 80 + 25t$
b. $\{t : 0 \leq t \leq 420\}$
c. $\{g(t) : 80 \leq g(t) \leq 10{,}580\}$
d. yes
e. No. The ratio $g(t) : t$ is not constant.

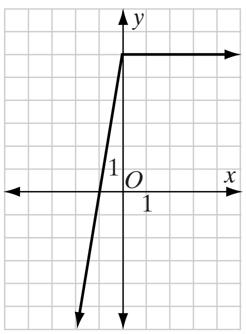
Hands-On Activity 1

1–2.



3. The graph of g(x) is the graph of f(x) reflected in the y-axis.
4. The graph of h(x) is the graph of f(x) reflected in the x-axis.

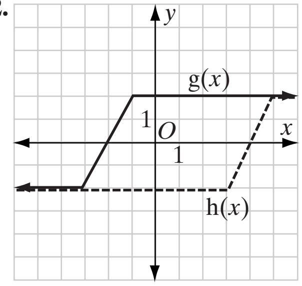
5.



6. The graph of p(x) is the graph of f(x) stretched vertically by a factor of 3.
7. The graph of $-$p(x) is the graph of f(x) reflected in the x-axis and stretched vertically by a factor of 3.
8. $f(x) = x + 1$
   (1) Graph of $g(x) = f(-x) = -x + 1$
   (2) Graph of $h(x) = -f(x) = -x - 1$
   (3) The graph of g(x) is the graph of f(x) reflected in the y-axis.
   (4) The graph of h(x) is the graph of f(x) reflected in the x-axis.
   (5) Graph of $3f(x) = 3x + 3$
   (6) The graph of p(x) is the graph of f(x) stretched vertically by a factor of 3.
   (7) The graph of $-$p(x) is the graph of f(x) reflected in the x-axis and stretched vertically by a factor of 3.

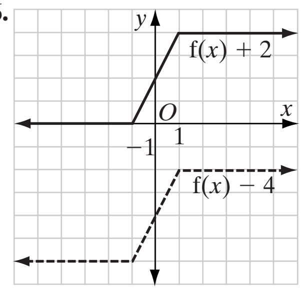
Hands-On Activity 2

1–2.



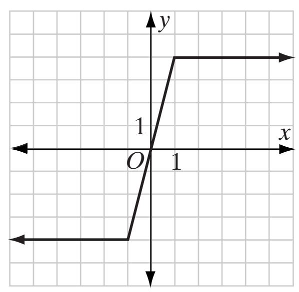
3. The graph of p(x) is the graph of f(x) shifted a units to the left.
4. The graph of p(x) is the graph of f(x) shifted a units to the right.

5–6.



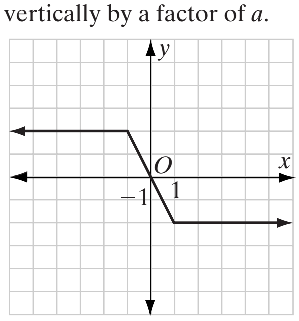
7. The graph of $f(x) + a$ is the graph of f(x) shifted a units up.
8. The graph of $f(x) - a$ is the graph of f(x) shifted a units down.

9.



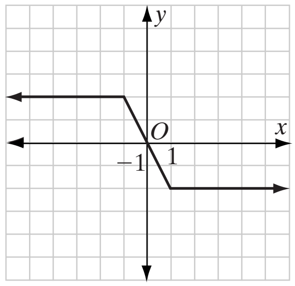
10. The graph of af(x) is the graph of f(x) stretched vertically by a factor of a.

11.



12. The graph of $-f(x)$ is the graph of $f(x)$ reflected in the x-axis or y-axis.

13.



14. The graph of $g(x)$ is the graph of $f(x)$ reflected in the x-axis or y-axis.

4-4 Absolute Value Functions *(pages 138–139)*

Writing About Mathematics

1. Yes. Each y then corresponds to exactly one x.

2. Yes. By definition, $f(x) = 2 - x$ when $x \leq 2$ and $f(x) = x - 2$ when $x > 2$.

Developing Skills

3. $(0, 0)$ **4.** $(-4, 0)$

5. $(14, 0)$ **6.** $(5, 0)$

In 7–10, the answer to part **a** is a graph.

7. b. $\{y : y \geq 0\}$ **8. b.** $\{y : y \geq 0\}$

9. b. $\{y : y \geq 1\}$ **10. b.** $\{y : y \geq -3\}$

Applying Skills

11. $h(x) = |2 - x|$

12. a. $m(x) = |x - 150|$

b. $h(x) = \frac{|x - 150|}{65}$

13. a–c. Graphs

d. The graph of $y = |x| + a$ is the graph of $y = |x|$ shifted vertically by the amount $|a|$. When $a > 0$, the shift is upward. When $a < 0$, the shift is downward.

14. a–c. Graphs

d. The graph of $y = |x + a|$ is the graph of $y = |x|$ shifted $|a|$ horizontally. When $a > 0$, the shift is to the left. When $a < 0$, the shift is to the right.

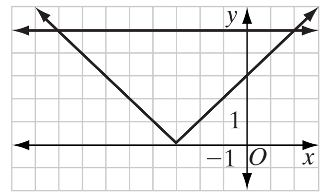
15. a–b. Graphs

c. The graph of $y = -|x|$ is the graph of $y = |x|$ reflected in the x-axis.

16. a–c. Graphs

d. The graph of $y = a|x|$ is the graph of $y = |x|$ stretched or compressed vertically. When $a > 0$, the graph is stretched vertically. When $a < 0$, the graph is compressed vertically.

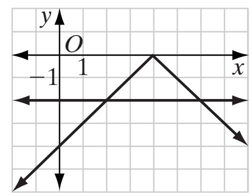
17. a.



b. $\{-8, 2\}$

c. $x < -8$ or $x > 2$

18. a.



d. $-8 < x < 2$

b. $\{2, 6\}$

c. $2 < x < 6$

d. $x < 2$ or $x > 6$

4-5 Polynomial Functions *(pages 147–149)*

Writing About Mathematics

1. Yes. Tiffany is correct. The graph never crosses or touches the x-axis.

2. This function has two roots; $x = 2$ is a double root.

Developing Skills

3. a. no real roots
b. $\{3\}$
c. no
d. no

4. a. -1
b. {all real numbers}
c. yes
d. yes

5. a. $\{0, 2\}$
b. $\{y : y \geq -1\}$
c. no
d. no

6. a. $\{-1, 3\}$
b. $\{y : y \leq 4\}$
c. no
d. no

7. a. 1
b. $\{y : y \leq 0\}$
c. no
d. no

8. a. 0
b. {all real numbers}
c. yes
d. yes

9. a. $\{-2, 0, 2\}$
b. {all real numbers}
c. no
d. no

10. a. $\{-3, -1, 1, 3\}$
b. $\{y : y \geq -1\}$
c. no
d. no

11. a. $\{-2, 0, 3\}$
b. {all real numbers}
c. yes
d. no

12. $-3 < x < 1$ **13.** $-3 \leq x \leq -1$

14. $x < -1$ or $x > 2$ **15.** { } or $\varnothing$

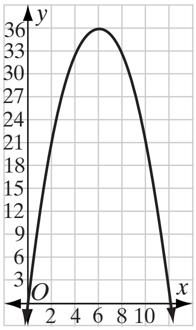
16. $x < 1$ or $x > 5$ **17.** $-4 \leq x \leq 1$

Applying Skills

18. a. $12 - x$

b. $y = x(12 - x)$ or $y = 12x - x^2$

c.

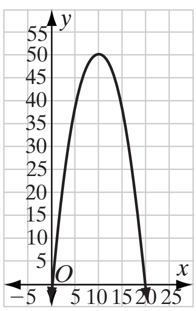


d. length = width = 6

19. a. $20 - x$

b. $y = \frac{1}{2}x(20 - x)$ or $y = 10x - \frac{x^2}{2}$

c.



d. Each leg measures 10 feet.

20. $p(x) = (x + 4)(x + 2)(x - 3)$ or $p(x) = x^3 + 3x^2 - 10x - 24$

21. a–c. Graphs

d. $y = x^2 + a$ is the graph of $y = x^2$ shifted vertically by the amount $|a|$. When $a > 0$, the graph of $y = x^2$ is shifted upward. When $a < 0$, the graph of $y = x^2$ is shifted downward.

e. $T_{0,a}$

22. a–c. Graph

d. $y = (x + a)^2$ is the graph of $y = x^2$ shifted horizontally by the amount $|a|$. When $a > 0$, the shift is to the left. When $a < 0$, the shift is to the right.

e. $T_{a,0}$

23. a–b. Graphs

c. The graph of $y = -x^2$ is the graph of $y = x^2$ reflected in the x-axis.

d. $R_{y=0}$

24. a–c. Graphs

d. $y = ax^2$ is the graph of $y = x^2$ stretched vertically.

e. $y = ax^2$ is the graph of $y = x^2$ compressed vertically.

25. $\left(-\frac{b}{2a}, \frac{4ac - b^2}{4a}\right)$

4-6 The Algebra of Functions

(pages 153–155)

Writing About Mathematics

1. No. $|2 - x|$ and $|x - 2|$ are both always ≥ 0 so their sum is always ≥ 0. Their sum is equal to 0 only when $x = 2$.

2. If $g(x) = x + 1$, $2g(x) = 2x + 2 \neq g(2x) = 2x + 1$. If $f(x) = x$, $2\,f(x) = 2x = f(2x) = 2x$.

Developing Skills

3. a. $\{0, 1, 2, 3, 4, 5\}$

b. $\{1, 2, 3, 4, 5, 6\}$

c. $\{1, 2, 3, 4, 5\}$

d. $\{(1, -3), (2, 1), (3, 7), (4, 15), (5, 25)\}$

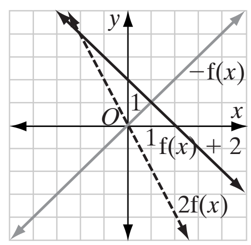
e. $\{1, 2, 3, 4\}$

f. $\left\{\left(1, \frac{1}{4}\right), \left(2, \frac{4}{3}\right), \left(3, \frac{9}{2}\right), (4, 16)\right\}$

4. a. $f(0) = 1$

b. $x = 1$

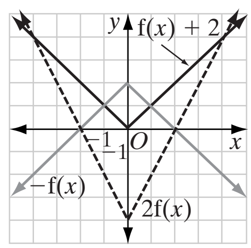
c–e.



5. a. $f(0) = -2$

b. $x = \pm 2$

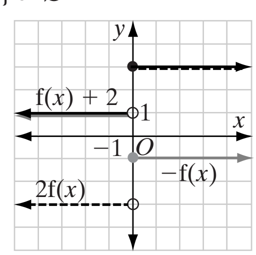
c–e.



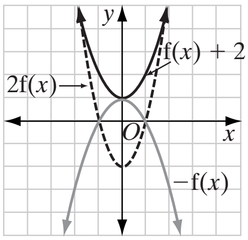
6. a. $f(0) = 1$

b. $\{\,\}$ or $\varnothing$

c–e.



7. a. $f(0) = -1$
b. $x = \pm 1$
c–e.



8. a. $x^2 - 2x + 4$
b. {all real numbers}

9. a. $x^2 - \frac{1}{x}$
b. $\{x : x \neq 0\}$

10. a. x
b. $\{x : x \neq 0\}$

11. a. $\frac{x^2}{4 - 2x}$
b. $\{x : x \neq 2\}$

12. a. $x^2 - 6x + 12$
b. {all real numbers}

13. a. $2x^3 - 4x^2$
b. {all real numbers}

Applying Skills

14. a. $c(x) = 10x + 2$
b. $t(x) = 0.15(10x) = 1.5x$
c. $e(x) = 10x + 2 + 0.15(10x) = 11.5x + 2$
d. e(3) = $36.50

15. a. $c(x) = 8.50x$
b. $s(x) = 0.50x + 2$
c. $t(x) = 8.50x + 0.50x + 2 = 9x + 2$
d. t(5) = $47.00

16. Answers will vary. Example:
Let $f(x) = x + 1$; then show $f(x) + f(x) = 2f(x)$.
$f(x) + f(x) = x + 1 + x + 1 = 2x + 2$
$2f(x) = 2(x + 1) = 2x + 2$
This result is true in general since doubling a function is the same as adding it to itself.

4-7 Composition of Functions
(pages 159–160)

Writing About Mathematics

1. Yes. $f(x) = x^2$ evaluates to $(a + 1)^2$ at $x = a + 1$ by definition.
2. $fg(x)$ is the product of the functions $f(x)$ and $g(x)$. $f(g(x))$ is function composition.

Developing Skills

3. 6 **4.** 10 **5.** −12 **6.** −8
7. 45 **8.** 1 **9.** −4 **10.** 0

In 11–18, the answer to part **d** is a graph.

11. a. $h(x) = 8x + 4$
b. {all real numbers}
c. {all real numbers}

12. a. $h(x) = 3x - 1$
b. {all real numbers}
c. {all real numbers}

13. a. $h(x) = 4 + x^2$
b. {all real numbers}
c. $\{y : y \geq 4\}$

14. a. $h(x) = x^2 + 8x + 16$
b. {all real numbers}
c. $\{y : y \geq 0\}$

15. a. $h(x) = x$
b. $\{x : x \geq 0\}$
c. $\{y : y \geq 0\}$

16. a. $h(x) = -|2 + x|$
b. {all real numbers}
c. $\{y : y \leq 0\}$

17. a. $h(x) = |5 - x|$
b. {all real numbers}
c. $\{y : y \geq 0\}$

18. a. $h(x) = x$
b. {all real numbers}
c. {all real numbers}

19. $f(g(x)) = |x + 3|$
$g(f(x)) = |x| + 3$

20. $f(g(x)) = |2x|$
$g(f(x)) = 2|x|$

21. $f(g(x)) = |2x + 3|$
$g(f(x)) = 2|x| + 3$

22. $f(g(x)) = |5 - x|$
$g(f(x)) = 5 - |x|$

23. Exercise 20: $g(x) = 2x$

24. $p(q(5)) = 2$
$q(p(5)) = 4$

25. Answers will vary. For example, $f(x) = 2x$ and $g(x) = x + 1$.

Applying Skills

26. a. $c(x) = 1.08x$
b. $d(x) = x - 10$
c. $c \circ d(x) = 1.08x - 10.80$
$d \circ c(x) = 1.08x - 10$
No. $c \circ d(x)$ applies sales tax to the discounted price, while $d \circ c(x)$ discounts the price after sales tax has been applied. It makes sense to use $c \circ d(x)$ on in-store discounts. It makes sense to use $d \circ c(x)$ on discounts applied after purchase, for example, a $10 mail-in rebate.
d. The function to use depends on whether the tax is applied to the full price or to the discounted price.

27. a. $n \circ f = 4(0.55x^2 + 1.66x + 50) - 160$
$= 2.2x^2 + 6.64x + 40$
b. 128.2 chirps per minute

4-8 Inverse Functions *(pages 166–167)*

Writing About Mathematics

1. Yes. The graph of the inverse is the reflection of the graph of f over the line $y = x$.
2. No. When the domain of an absolute value function is the set of real numbers, the function is not one-to-one and has no inverse function.

Developing Skills

3. 3 **4.** 5 **5.** -2 **6.** 8
7. 12 **8.** -6 **9.** -6 **10.** $\sqrt{2}$

11. Yes; $f^{-1} = \{(8, 0), (7, 1), (6, 2), (5, 3), (4, 4)\}$

12. Yes; $f^{-1} = \{(4, 1), (7, 2), (10, 1), (13, 4)\}$

13. Yes; $f^{-1} = \{(8, 0), (6, 2), (4, 4), (2, 6)\}$

14. No. The function is not one-to-one, so it has no inverse function.

15. No. This relation is not a function.

16. Yes; $f^{-1} = \{(x, y): y = \sqrt{x - 2} \text{ for } 2 \leq x \leq 27\}$

17. a. $f^{-1}(x) = \frac{x + 3}{4}$
b. domain of f = domain of f^{-1} = {all real numbers}
range of f = range of f^{-1} = {all real numbers}

18. a. $g^{-1}(x) = x + 5$
b. domain of g = domain of g^{-1} = {all real numbers}
range of g = range of g^{-1} = {all real numbers}

19. a. $f^{-1}(x) = 3x - 5$
b. domain of f = domain of f^{-1} = {all real numbers}
range of f = range of f^{-1} = {all real numbers}

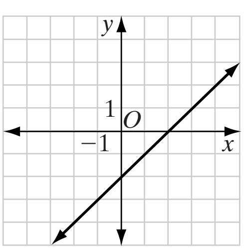
20. a. $f^{-1} = x^2$
b. domain of f = domain of $f^{-1} = \{x : x \geq 0\}$
range of f = range of $f^{-1} = \{y : y \geq 0\}$

21. $f^{-1} = \left\{(x, y) : y = \frac{x}{5}\right\}$

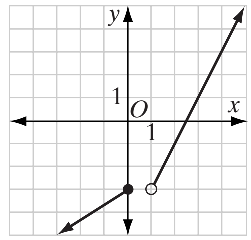
22. $g^{-1}(x) = 7 - x$. Yes, a function can be its own inverse.

23. No. $y = x^2$ is not one-to-one if the domain is the set of real numbers.

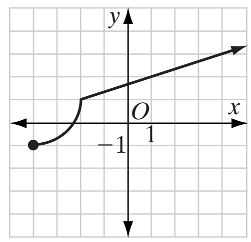
24.



25.



26.



Applying Skills

27. $f^{-1} = \frac{x}{0.2532}$
$f(f^{-1}(x)) = 0.2532\left(\frac{x}{0.2532}\right) = x$
$f^{-1}(f(x)) = \frac{0.2532x}{0.2532} = x$
Since $f(f^{-1}(x)) = f^{-1}(f(x)) = x$, the functions are inverses.

28. a. Graph
b. domain $= \{x : x \geq -4\}$
range $= \{y : y \geq 2\}$
c. domain $= \{x : x \geq 2\}$
range $= \{y : y \geq -4\}$
d. The domain of the function is the range of the inverse and the range of the function is the domain of the inverse.

4-9 Circles *(pages 172–173)*

Writing About Mathematics

1. No. A circle does not pass the vertical line test.
2. In center-radius form, the constant term is the square of the radius, and this cannot be negative.

Developing Skills

3. a. $x^2 + y^2 = 4$
b. $x^2 + y^2 - 4 = 0$

4. a. $x^2 + y^2 = 9$
b. $x^2 + y^2 - 9 = 0$

5. a. $x^2 + y^2 = 16$
b. $x^2 + y^2 - 16 = 0$

6. a. $(x - 4)^2 + (y - 2)^2 = 1$
b. $x^2 + y^2 - 8x - 4y + 19 = 0$

7. a. $(x + 1)^2 + (y - 1)^2 = 16$
b. $x^2 + y^2 + 2x - 2y - 14 = 0$

8. a. $(x - 6)^2 + (y - 5)^2 = 100$
b. $x^2 + y^2 - 12x - 10y - 39 = 0$
9. a. $(x - 6)^2 + (y - 13)^2 = 169$
b. $x^2 + y^2 - 12x - 26y + 36 = 0$
10. a. $x^2 + (y - 1)^2 = 17$
b. $x^2 + y^2 - 2y - 16 = 0$
11. $x^2 + y^2 = 16$
12. $(x - 2)^2 + (y - 3)^2 = 1$
13. $(x - 1)^2 + (y + 1)^2 = 9$
14. $(x + 2)^2 + (y - 3)^2 = 4$
15. $(x - 1)^2 + (y + 1)^2 = 25$
16. $x^2 + (y + 1)^2 = 4$
17. $(x + 1)^2 + (y - 3)^2 = 9$
18. $(x - 1)^2 + (y - 1)^2 = 13$
19. $(x + 1)^2 + (y + 1)^2 = 13$
20. a. $x^2 + y^2 = 25$
b. $(0, 0)$
c. 5
21. a. $(x - 1)^2 + (y - 1)^2 = 9$
b. $(1, 1)$
c. 3
22. a. $(x + 1)^2 + (y - 2)^2 = 4$
b. $(-1, 2)$
c. 2
23. a. $(x - 3)^2 + (y + 1)^2 = 16$
b. $(3, -1)$
c. 4
24. a. $(x + 3)^2 + (y - 3)^2 = 12$
b. $(-3, 3)$
c. $2\sqrt{3}$
25. a. $x^2 + (y - 4)^2 = 16$
b. $(0, 4)$
c. 4
26. a. $(x + 5)^2 + (y - 2.5)^2 = 63.25$
b. $(-5, 2.5)$
c. $\sqrt{63.25} = \frac{\sqrt{253}}{2}$
27. a. $\left(x + \frac{1}{2}\right)^2 + \left(y - \frac{3}{2}\right)^2 = \frac{18}{4}$
b. $\left(-\frac{1}{2}, \frac{3}{2}\right)$
c. $\frac{3}{2}\sqrt{2}$

Applying Skills

28. a. Graph
b. Yes. The cube can easily pass through the arch because its sides are shorter than the radius of the arch.
c. Yes. If the prism is placed in the center of the arch so that its base is 8 feet, it will have just under 7 feet of clearance to pass under the arch.
29. $10\sqrt{2}$
30. Width $= 4\sqrt{5}$, length $= 8\sqrt{5}$
31. Answers will vary: any three equations of the form $(x - 2)^2 + (y - 2)^2 = r^2$ with three different positive values for r.

Hands-On Activity

1. $(1, 3)$
2. Slope of $\overline{PQ} = 0$; the slope of the line perpendicular to $\overline{PQ}$ is undefined.
3. $x = 1$
4. (1) $(0, 0)$
(2) Slope of $\overline{QR} = -1$; slope of the line perpendicular to $\overline{QR} = 1$
(3) $y = x$
5. $C(1, 1)$
6. $CP = CQ = CR = 2\sqrt{5}$
7. Equation: $(x - 1)^2 + (y - 1)^2 = 20$
P is on the circle: $(5 - 1)^2 + (3 - 1)^2 \stackrel{?}{=} 20$
$16 + 4 = 20$ ✔
Q is on the circle: $(-3 - 1)^2 + (3 - 1)^2 \stackrel{?}{=} 20$
$16 + 4 = 20$ ✔
R is on the circle: $(3 - 1)^2 + (-3 - 1)^2 \stackrel{?}{=} 20$
$4 + 16 = 20$ ✔

4-10 Inverse Variation *(pages 177–178)*

Writing About Mathematics

1. No. The function f cannot be represented in the form $f(x) = y = a_nx^n + a_{n-1}x^{n-1} + \cdots + a_0$.
2. In direct variation, both quantities increase or decrease by the same factor. In inverse variation, as one quantity increases by a factor a, the other quantity decreases by the factor $\frac{1}{a}$.

Developing Skills

3. $xy = 2$ or $y = \frac{2}{x}$
4. $xy = 6$ or $y = \frac{6}{x}$
5. $xy = -8$ or $y = \frac{-8}{x}$
6. inversely
7. directly
8. directly
9. directly
10. inversely
11. neither directly nor indirectly
12. directly

Applying Skills

13. The width of rectangle $ABCD$ is equal to half the width of rectangle $EFGH$.
14. He can ride to school in one-third the time it takes him to walk.
15. a. Yes. $D = RT$. As rate increases, time traveled decreases, when distance is constant.
b. 45 mph

16. Initial trip: 55 mph for 3 hrs
Return trip: 33 mph for 5 hrs
17. 1st time: 16 cans at \$1.50 per can
2nd time: 15 cans at \$1.60 per can

Review Exercises *(pages 180–183)*

1. Not a function since (1, 1) and (1, −1) have the same first element.
2. Function. Each x of the domain has only one y-value.
3. Not a function since for most values of x in the domain there are two distinct y-values in the range.
4. Function. Each x value of the domain has only one y-value in the range.
5. Function. Each x value of the domain has only one y-value in the range.

6. a. yes **b.** yes **c.** yes; $y = \frac{x-3}{4}$
7. a. no **b.** no **c.** no
8. a. no **b.** no **c.** no
9. a. yes **b.** yes **c.** yes; $y = \sqrt[3]{x}$
10. a. no **b.** no **c.** no
11. a. yes **b.** yes **c.** yes; $y = x^2 + 4, x \geq 0$
12. 1
13. $\{-1, 3\}$
14. $\{-2, 4\}$
15. $\{-2, -1, 1\}$
16. 1
17. $\{-3, 5\}$
18. $x = -7$ and $x = 1$
19. $\{(0, 7), (1, 8), (2, 9), (3, 10), (4, 11)\}$
20. $\{(0, 5), (1, 2), (2, -1), (3, -4), (4, -7)\}$
21. $\{(0, 6), (1, 15), (2, 20), (3, 21), (4, 18)\}$
22. $\left\{(0, 6), \left(1, \frac{5}{3}\right), \left(2, \frac{4}{5}\right), \left(3, \frac{3}{7}\right), \left(4, \frac{2}{9}\right)\right\}$
23. $f(g(x)) = 2x + 6$; $g(f(x)) = 2x + 3$
24. $f(g(x)) = |4x - 1|$; $g(f(x)) = 4|x| - 1$
25. $f(g(x)) = (x + 2)^2$; $g(f(x)) = x^2 + 2$
26. $f(g(x)) = \sqrt{5x - 3}$; $g(f(x)) = 5\sqrt{x} - 3$
27. $f(g(x)) = \frac{2 + 3x}{2}$; $g(f(x)) = 2 + \frac{3}{2}x$
28. $f(g(x)) = \frac{1}{2x}$; $g(f(x)) = \frac{50}{x}$
29. $f(g(x)) = x$; $g(f(x)) = x$
30. $f(g(x)) = x$; $g(f(x)) = x$
31. $x^2 + y^2 = 9$
32. $(x - 3)^2 + (y - 3)^2 = 9$
33. $(x - 3)^2 + y^2 = 25$
34. $x^2 + (y + 1)^2 = 16$
35. $(x + 1)^2 + (y + 1)^2 = 5$
36. $(x + 2)^2 + (y - 2)^2 = 8$
37. $(-1, -2)$ and $(3, 2)$

Check $(-1, -2)$:
$$(x - 2)^2 + (y + 1)^2 = 10$$
$$(-3)^2 + (-1)^2 \stackrel{?}{=} 10$$
$$10 = 10 \checkmark$$
$$y = x - 1$$
$$-2 \stackrel{?}{=} -1 - 1$$
$$-2 = -2 \checkmark$$
Check $(3, 2)$:
$$(x - 2)^2 + (y + 1)^2 = 10$$
$$(1)^2 + (3)^2 \stackrel{?}{=} 10$$
$$10 = 10 \checkmark$$
$$y = x - 1$$
$$2 \stackrel{?}{=} 3 - 1$$
$$2 = 2 \checkmark$$

38. $y = (x - 4)^2 - 2$
39. $y = 3|x|$
40. $y = -2x - 3$
41. $y = -|x + 1| + 3$
42. a. $f^{-1}(x) = \frac{1}{2}(x - 8)$ **b.** Yes, since f is one-to-one.
43. a. $f^{-1}(x) = \frac{3}{x}$ **b.** Yes, since f is one-to-one.
44. a. $f^{-1}(x) = \sqrt{x}$ **b.** No, f is not one-to-one. **c.** $\{x : x \geq 0\}$
45. a. $f^{-1}(x) = x^2$ **b.** Yes, since f is one-to-one.

Exploration *(page 183)*

1. The base is a circle. The two cut edges are circles.
2. The cut surfaces are ellipses.
3. The curved portion of the edges is a parabola.
4. The shape is a hyperbola.
5. The shape is a pair of intersecting lines.

Cumulative Review *(pages 184–185)*

Part I

1. 3 **2.** 2 **3.** 2
4. 2 **5.** 3 **6.** 4
7. 4 **8.** 2 **9.** 4
10. 4

Part II

11. $2(x + 1)^2(x - 1)$
12. $\frac{3 + \sqrt{5}}{3 - \sqrt{5}} \cdot \frac{3 + \sqrt{5}}{3 + \sqrt{5}} = \frac{(3 + \sqrt{5})^2}{9 - 5} = \frac{7 + 3\sqrt{5}}{2}$

Part III

13. *Answer:* $-3 < x < 5$

$$x^2 - 2x - 15 < 0$$
$$(x + 3)(x - 5) < 0$$

Let $(x + 3) < 0$:

$$x + 3 < 0 \quad \big| \quad x - 5 > 0$$
$$x < -3 \quad \big| \quad x > 5$$

Solution: { }

Let $(x + 3) > 0$:

$$x + 3 > 0 \quad \big| \quad x - 5 < 0$$
$$x > -3 \quad \big| \quad x < 5$$

Solution: $-3 < x < 5$

Combine the two solutions.

14. $D = -2h = -4, E = -2k = 6$

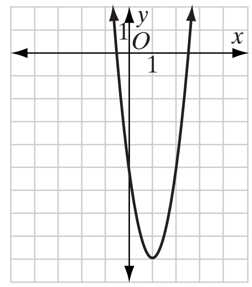
$F = h^2 + k^2 - r^2 = 4 + 9 - 16 = -3$

$x^2 + y^2 + Dx + Ey + F = 0$

$x^2 + y^2 - 4x + 6y - 3 = 0$

Part IV

15. a.



b. $x = 1$

c. $(1, -9)$

d. $x = -0.5$ and $x = 2.5$

16. $\dfrac{2 + \frac{2}{a}}{1 - \frac{1}{a^2}} \cdot \dfrac{a^2}{a^2} = \dfrac{2a^2 + 2a}{a^2 - 1} = \dfrac{2a(a + 1)}{(a + 1)(a - 1)} = \dfrac{2a}{a - 1}$;

undefined for $a = 0, a = \pm 1$.

Chapter 5. Quadratic Functions and Complex Numbers

5-1 Real Roots of a Quadratic Equation

(pages 192–193)

Writing About Mathematics

1. $0 = 2x^2 - x - 1$

$0 = 16x^2 - 8x - 8$

$0 = 8(x - 1)(2x + 1)$

$x = 1, -\frac{1}{2}$

2. Yes. The resulting equation is equivalent to the original. The new equation can be solved by completing the square.

Developing Skills

3. $+9 = (x + 3)^2$

4. $+16 = (x - 4)^2$

5. $+1 = (x - 1)^2$

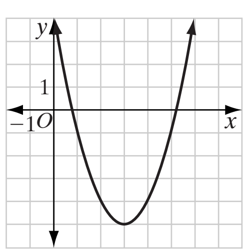
6. $+36 = (x - 6)^2$

7. $+2 = 2(x - 1)^2$

8. $+\frac{9}{4} = \left(x - \frac{3}{2}\right)^2$

In 9–14, part **b**, answers will vary.

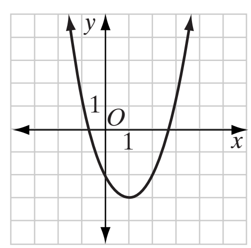
9. a.



b. 0.8, 5.2

c. $3 \pm \sqrt{5}$

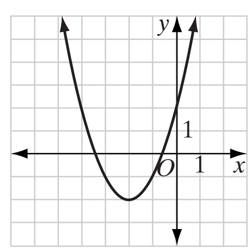
10. a.



b. −0.7, 2.7

c. $1 \pm \sqrt{3}$

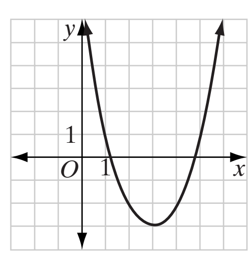
11. a.



b. −0.6, −3.4

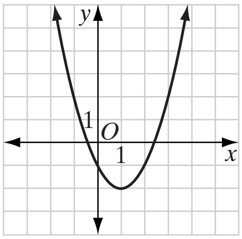
c. $-2 \pm \sqrt{2}$

12. a.

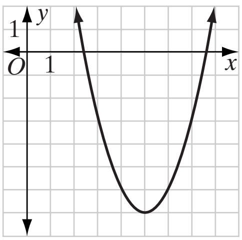


b. 1.3, 4.7

c. $3 \pm \sqrt{3}$

13. a. 

b. $-0.4, 2.4$
c. $1 \pm \sqrt{2}$

14. a. 

b. $2.4, 7.7$
c. $5 \pm \sqrt{7}$

15. $1 \pm \sqrt{3}$
16. $-3 \pm \sqrt{5}$
17. $2 \pm \sqrt{3}$
18. $-1 \pm \sqrt{6}$
19. $3 \pm \sqrt{7}$
20. $4 \pm 2\sqrt{3}$
21. $-3 \pm \sqrt{\frac{15}{2}}$
22. $1 \pm \frac{2\sqrt{3}}{3}$
23. $\frac{3 \pm \sqrt{3}}{2}$
24. $\left\{\frac{1}{2}, \frac{9}{2}\right\}$
25. $-1 \pm \sqrt{7}$
26. $\frac{-3 \pm \sqrt{33}}{2}$
27. a. $\frac{5 \pm \sqrt{21}}{2}$
b. $0.2, 4.8$
28. translated 6 right, 31 down
29. translated 1 left, 3 down
30. translated 3 right, 16 down
31. translated $\frac{1}{2}$ left, 2 up
32. reflected about x-axis, translated $\frac{1}{2}$ right, $2\frac{1}{4}$ or $\frac{9}{4}$ up
33. stretched vertically by a factor of 3, translated 1 unit left
34. Vertex: $(-4, -11)$, axis of symmetry: $x = -4$
Complete the square to get $f(x) = (x + 4)^2 - 11$.

Applying Skills

35. a. Width $= -1 + 2\sqrt{5}$ ft, length $= 2 + 4\sqrt{5}$ ft
b. $(-1 + 2\sqrt{5})(2 + 4\sqrt{5})$
$= 2(2\sqrt{5} + 1)(2\sqrt{5} - 1) = 38 \text{ ft}^2$
c. Width = 3.5 ft, length = 10.9 ft
36. a. Base 1 $= 2\sqrt{6} - 2$ ft, base 2 $= 2\sqrt{6} + 6$ ft, height $= 2\sqrt{6} - 2$ ft
b. $\left(\frac{2\sqrt{6} - 2}{2}\right)((2\sqrt{6} - 2) + (2\sqrt{6}))$
$= (\sqrt{6} - 1)(4\sqrt{6} + 4) = 20 \text{ ft}^2$
c. Base 1 = height = 2.9 ft, base 2 = 10.9 ft
37. Steve is 13, Alice is 15. Use $x(x + 2) = 195$.

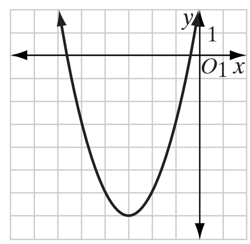
5-2 The Quadratic Formula *(pages 195–197)*

Writing About Mathematics

1. No. The denominator applies to all the terms in the numerator.
2. Yes. When $b^2 < 4ac$, the roots involve the square root of a negative number, which is not real.

Developing Skills

3. $-1, -4$
4. $-7, 1$
5. $\frac{3 \pm \sqrt{5}}{2}$
6. $\frac{1 \pm \sqrt{17}}{2}$
7. $\frac{-5 \pm \sqrt{33}}{2}$
8. $\pm 2\sqrt{2}$
9. $0, 3$
10. $-1 \pm \sqrt{5}$
11. $\frac{2}{3}, 1$
12. $\frac{1 \pm \sqrt{17}}{8}$
13. $\frac{5 \pm \sqrt{33}}{4}$
14. $\frac{1 \pm \sqrt{33}}{4}$
15. $3 \pm \sqrt{6}$
16. $\frac{1}{2} \pm \sqrt{3}$
17. $\frac{2 \pm \sqrt{10}}{3}$

18. a. 

b. Answers will vary: $-0.4, -5.6$
c. $-3 \pm \sqrt{7}$
d. $-0.4, -5.6$

Applying Skills

19. $1 + \sqrt{6}, 7 + 2\sqrt{6}$ or $1 - \sqrt{6}, 7 - 2\sqrt{6}$
20. Width $= -1 + \sqrt{3}$ ft, length $= 1 + \sqrt{3}$ ft
21. Width $= -2 + \sqrt{46}$ cm, length $= 2 + \sqrt{46}$ cm
22. Altitude $= -3 + 3\sqrt{5}$ ft, base $= 3 + 3\sqrt{5}$ ft
23. Bases = 8, 12; height = 4
24. $DB = -2 + 2\sqrt{37}, AD = 2 + 2\sqrt{37}, AB = 4\sqrt{37}$
25. a. $\left(\frac{-b - \sqrt{b^2 - 4ac}}{2a}, 0\right)$ and $\left(\frac{-b + \sqrt{b^2 - 4ac}}{2a}, 0\right)$
b. $\left(\frac{-b}{2a}, 0\right)$
c. $x = \frac{-b}{2a}$
d. $\frac{-b}{2a}$
26. 11.9 seconds
27. a. 1.2
b. 2.3
c. 16.7
28. $x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = \frac{b^2}{4a^2} - \frac{c}{a}$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The roots are the same.

Hands-On Activity:
Alternate Derivation of the Quadratic Formula

Yes.
$$2ax + b = \pm\sqrt{b^2 - 4ac}$$
$$2ax = -b \pm \sqrt{b^2 - 4ac}$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

5-3 The Discriminant *(pages 201–203)*

Writing About Mathematics

1. **a.** 9
   b. $\frac{-\sqrt{5} \pm 3}{2}$
   c. No. The rules apply only when *a, b,* and *c* are rational numbers.
2. Yes. Since b^2 is always positive, when $-4ac$ is positive, $b^2 - 4ac > 0$.

Developing Skills

3. < 0	**4.** > 0	**5.** < 0
6. $= 0$	**7.** > 0	**8.** $= 0$

9. **a.** rational and unequal
   b. 2
10. **a.** irrational and unequal
   b. 2
11. **a.** rational and equal
   b. 1
12. **a.** irrational and unequal
   b. 2
13. **a.** not real numbers
   b. 0
14. **a.** rational and unequal
   b. 2
15. **a.** 0, rational and equal
   b. 6
16. **a.** 49, rational and unequal
   b. $0, -\frac{7}{2}$
17. **a.** 5, irrational and unequal
   b. $\frac{-3 \pm \sqrt{5}}{2}$
18. **a.** 64, rational and unequal
   b. ± 2
19. **a.** 17, irrational and unequal
   b. $\frac{1 \pm \sqrt{17}}{8}$
20. **a.** -11, not real numbers
   b. no real roots
21. **a.** 0, rational and equal
   b. $\frac{1}{2}$
22. **a.** 49, rational and unequal
   b. $-1, \frac{5}{2}$
23. **a.** -11, irrational and unequal
   b. no real roots
24. Yes. A perfect square trinomial is the only way to yield equal rational roots.
25. **a.** $c = 1$
   b. any $c < 1$ such that $4 - 4c$ a perfect square
   c. any $c < 1$ such that $4 - 4c$ is not a perfect square
   d. $c > 1$
26. **a.** ± 4
   b. any $b < -4$ or $b > 4$ such that $b^2 - 16$ is a perfect square
   c. any $b < -4$ or $b > 4$ such that $b^2 - 16$ is not a perfect square
   d. $-4 < b < 4$

Applying Skills

27. The fence cannot be constructed. Use $x^2 + (15 - x)^2 = 8^2$. The discriminant is -188, so the equation has no real roots.
28. Yes. Use $4x(5 - x) = 25$. The discriminant is 0.
29. Yes. Use $-16x^2 + 48x = 32$. The determinant is 256.
30. No. That value for the profit yields a negative determinant.

5-4 The Complex Numbers *(pages 208–209)*

Hands-On Activity

For the parallelogram with vertices $4 + 2i$, $2 - 5i$, and 0, the fourth vertex is $6 - 3i$, which is the sum of the two given complex numbers.

In 1–9, the resulting complex number is always the sum of the two complex numbers. Student answers should include graphs of parallelograms on the complex plane.

1. $5 + 5i$	**2.** $-7 + 7i$	**3.** $6 - 4i$
4. $-1 - 7i$	**5.** 2	**6.** -10
7. $-3 - 4i$	**8.** $3i$	**9.** $4 + 2i$

Writing About Mathematics

1. No. Factoring out *i* from each term and then multiplying yields the product -4.
2. Yes. $i \cdot i = i^2 = -1$ and any real coefficient, when squared, is positive.

Developing Skills

3. $2i$	**4.** $9i$
5. $3i$	**6.** $-6i$
7. $-11i$	**8.** $2i\sqrt{2}$
9. $2i\sqrt{3}$	**10.** $-6i\sqrt{2}$
11. $15i\sqrt{3}$	**12.** $-2i\sqrt{5}$
13. $-i\sqrt{51}$	**14.** $10i\sqrt{5}$
15. $5 + i\sqrt{5}$	**16.** $1 + i\sqrt{3}$
17. $-4 - 2i\sqrt{6}$	**18.** $-3 + 6i$
19. $19i$	**20.** $3i$
21. $13i$	**22.** $3i$
23. 0	**24.** $7i\sqrt{5}$

25. $-4 + 8i$ **26.** $15\sqrt{2} + 6i\sqrt{2}$
27. $2 + 6i\sqrt{2}$ **28.** $-4 + i\sqrt{7}$
29. $-1 - 4i\sqrt{10}$ **30.** $5 - 3i$
31. $-3 + 7i$ **32.** $\frac{3}{7} - \frac{i\sqrt{2}}{4}$
33. $-1 + i\sqrt{6}$ **34.** $0.45 + 0.3i$
35. i **36.** -3
37. $6i$ **38.** $-5i$
39. i **40.** $7 - 2i$
41. 1 **42.** 0
43. i **44.** $(2, 4)$
45. $(-2, 5)$ **46.** $(4, -2)$
47. $(-4, -2)$ **48.** $\left(\frac{1}{2}, 4\right)$
49. $(0, 3)$ **50.** $(-3, 0)$
51. $(0, 0)$

Applying Skills

52. $13i$ ohms **53.** $-12i$ ohms

Hands-On Activity: Multiplying Complex Numbers

Multiplication by i

1. $-3 + 2i$ **2.** -12
3. $\frac{-3}{2} + \frac{1}{2}i$ **4.** 5
5. $-10i$ **6.** $2 - 3i$

Multiplication by a real number

1. $8 + 16i$ **2.** $\frac{-5}{2} - \frac{5}{2}i$
3. $3 + 8i$ **4.** $4i$
5. $-2i$ **6.** $6 - 9i$

5-5 Operations With Complex Numbers *(pages 215–216)*

Writing About Mathematics

1. Yes, $i^2 = -1$.
2. Yes, $(a + bi)(a - bi) = a^2 - b^2i^2 = a^2 + b^2$.

Developing Skills

3. $7 + 9i$ **4.** $5 - 4i$
5. $9 - 4i$ **6.** $-4 - 2i$
7. $-16i$ **8.** $10i$
9. $7i$ **10.** $-2 - 19i$
11. 0 **12.** $\frac{3}{4} - \frac{1}{4}i$
13. $\frac{8}{3} - \frac{7}{6}i$ **14.** $\frac{4}{5} + \frac{2}{5}i$
15. $\frac{-15}{4} + \frac{28}{5}i$ **16.** $\frac{29}{4} + \frac{121}{10}i$
17. $\frac{25}{42} - \frac{13}{24}i$ **18.** $3 - 4i$
19. $2 + 5i$ **20.** $-8 - i$
21. $-6 + 9i$ **22.** $\frac{1}{2} + 3i$
23. $-4 - \frac{1}{3}i$ **24.** $\frac{5}{3} + \frac{2}{3}i$
25. $\pi - 2i$ **26.** $-9 + 7i$
27. $3 + 15i$ **28.** $-5 - 12i$
29. $2 - 23i$ **30.** $11 + 23i$
31. $-5i$ **32.** 17
33. -148 **34.** $34i$
35. 1 **36.** 1
37. -1 **38.** $\frac{1}{2} - \frac{1}{2}i$
39. $\frac{1}{10} - \frac{1}{5}i$ **40.** $-\frac{1}{5} - \frac{2}{5}i$
41. $\frac{1}{6} + \frac{1}{6}i$ **42.** $\frac{8}{5} + \frac{4}{5}i$
43. $\frac{8}{17} + \frac{2}{17}i$ **44.** $\frac{30}{349} - \frac{108}{349}i$
45. $\frac{9}{81 + \pi^2} - \frac{\pi}{81 + \pi^2}i$ **46.** $6 - 2i$
47. $-1 + 3i$ **48.** $4 - 3i$
49. $3 - 4i$ **50.** $\frac{1}{5} + \frac{3}{5}i$
51. $-\frac{1}{4} - \frac{7}{4}i$ **52.** $\frac{7}{5} - \frac{11}{5}i$
53. $\frac{4}{5} + \frac{3}{5}i$ **54.** $\frac{21}{29} - \frac{20}{29}i$
55. $1 - 4i$ **56.** $3 - 4i$
57. $\frac{7}{125} + \frac{1}{125}i$ **58.** $2i$
59. $\frac{6}{4\pi^2 + 1} + \frac{12\pi}{4\pi^2 + 1}i$ **60.** $-\frac{1}{5} + \frac{1}{35}i$

Applying Skills

61. $\frac{4}{3} + 6i$ **62.** $\frac{-1}{5}i$

5-6 Complex Roots of a Quadratic Equation *(page 219)*

Writing About Mathematics

1. Yes. $b^2 - 4ac$ will be negative, and since $b = 0$, there will be no real component.
2. Yes. The two roots are made by adding and subtracting the imaginary component from the same real component.

Developing Skills

3. $2 \pm 2i$ **4.** $-3 \pm i$
5. $2 \pm 3i$ **6.** $\frac{-1}{2} \pm \frac{1}{2}i$
7. $-5 \pm 2i$ **8.** $-4 \pm i$
9. $1 \pm 3i$ **10.** $\frac{1}{2} \pm i$
11. $-\frac{1}{2} \pm 2i$ **12.** $2 \pm i$
13. $2 \pm i\sqrt{3}$ **14.** $1 \pm i\sqrt{2}$

5-7 Sum and Product of the Roots of a Quadratic Equation *(pages 223–224)*

Writing About Mathematics

1. $x^2 - 2px + p^2 - q = 0$
2. Both. Olivia's equation is Adrien's multiplied by 2.

Developing Skills

3. Sum = -1, Product = 1 **4.** Sum = -4, Product = 5

5. Sum = $\frac{3}{2}$
Product = -1

6. Sum = $-\frac{2}{5}$
Product = -2

7. Sum = 2
Product = $\frac{4}{3}$

8. Sum = 3
Product = -1

9. Sum = 8
Product = -12

10. Sum = $\frac{1}{2}$
Product = $-\frac{9}{4}$

11. Sum = $\frac{5}{2}$
Product = -4

12. Sum = 0
Product = $-\frac{1}{4}$

13. Sum = 0
Product = 1

14. Sum = -2
Product = 0

15. Sum = $-\frac{3}{4}$
Product = $-\frac{9}{8}$

16. Sum = -2
Product = -3

17. Sum = -1
Product = $\frac{5}{3}$

18. -10 **19.** 11 **20.** $-\frac{5}{2}$

21. $-\frac{13}{4}$ **22.** $-\frac{1}{6}$ **23.** $\frac{5}{6}$

24. 1 **25.** $-\frac{11}{4}$ **26.** 3

27. $-\frac{5}{7}$

28. **a.** $3 - \sqrt{2}$
b. $6 - 9\sqrt{2}$
c. The coefficients of the equation are not rational numbers.

29. **a.** $-11 - \sqrt{3}$
b. $3 + 11\sqrt{3}$
c. The coefficients of the equation are not rational numbers.

30. $x^2 - 7x + 10 = 0$
31. $x^2 - 11x + 28 = 0$
32. $x^2 - x - 12 = 0$
33. $x^2 + 3x + 2 = 0$
34. $x^2 - 9 = 0$
35. $4x^2 - 16x + 7 = 0$
36. $32x^2 - 12x - 9 = 0$
37. $x^2 - x = 0$
38. $x^2 - 4x + 1 = 0$
39. $x^2 - x - 1 = 0$
40. $9x^2 + 6x - 2 = 0$
41. $x^2 - 6x + 10 = 0$
42. $4x^2 - 12x + 13 = 0$
43. $4x^2 + 9 = 0$

Applying Skills

44. $x^2 - 15x + 54 = 0$
45. $x^2 - 12x + 40 = 0$
46. Answers will vary. Correct as long as $-b = c$.
Example: $x^2 - 4x + 4 = 0$
47. Sum:

$$\frac{-b + \sqrt{b^2 - 4ac}}{2a} + \frac{-b - \sqrt{b^2 - 4ac}}{2a} = \frac{-2b}{2a} = \frac{-b}{a}$$

Product:

$$\frac{-b + \sqrt{b^2 - 4ac}}{2a} \cdot \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{b^2 - (b^2 - 4ac)}{4a^2} = \frac{c}{a}$$

48. c is the product of the roots. Since $-b$ is an integer equal to the sum of the roots and one root is an integer, both roots are integers. Therefore, both roots are factors of c.

5-8 Solving Higher Degree Polynomial Equations *(pages 227–228)*

Writing About Mathematics

1. Yes. This follows from the definition of a root.
2. Yes. f$(x - a)$ is a translation of f $|a|$ units to the right. Thus, each root is increased by $|a|$.

Developing Skills

3. $0, -2, -5$
4. $0, 1, -2$
5. $-3, \pm 2i$
6. $1, \pm i\sqrt{3}$
7. $\frac{3}{2}, \pm 1$
8. $3, \pm\frac{\sqrt{2}}{2}i$
9. $\pm 1, \pm 2$
10. $\pm i, \pm 2i$
11. $\pm 3, \pm 3i$
12. $\pm\frac{1}{2}, \pm\frac{1}{2}i$
13. $\pm 1, \pm 3$
14. $0, -1, 2$
15. $0, \pm 3\sqrt{2}$
16. $-1, \frac{1}{2}, \frac{3 \pm i\sqrt{7}}{2}$
17. $\pm 1, \frac{-1 \pm i\sqrt{2}}{3}$
18. $-2, \pm 1$
19. **a.** 0 **b.** Yes
20. **a.** -16 **b.** No
21. **a.** 2 **b.** No
22. **a.** 0 **b.** Yes
23. **a.** 1 **b.** No
24. **a.** 0 **b.** Yes
25. **a.** 0 **b.** Yes
26. **a.** $3 + \sqrt{3}$ **b.** No
27. **a.** $1 + 9i$ **b.** No
28. **a.** $\frac{-33}{4} + \frac{159}{8}i$ **b.** No

Applying Skills

29. **a.** Multiply out to check.
b. $1, -\frac{1}{2} \pm \frac{\sqrt{3}}{2}i$
c. Same as part **b**. The two equations are equal, so they have equal roots.
d. Prove by multiplying.

30. **a.** Multiply out to check.
b. $-1, \frac{1}{2} \pm \frac{\sqrt{3}}{2}i$
c. Same as part **b**. The two equations are equal, so they have equal roots.
d. Prove by multiplying.

31. **a.** The graph of g(x) is that of f(x) stretched vertically by a factor of 2.
b. They are the same.
c. The graph of p(x) will be that of q(x) stretched vertically by a factor of a.
d. They are the same.

Hands-On Activity

a. $-1, 1, 2$
b. $-2, 2, 3$
c. $-2, -1, 3$
d. $-2, -1, 1, 2$

5-9 Solutions of Systems of Equations and Inequalities *(pages 236–239)*

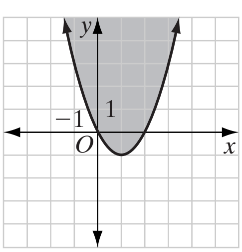
Writing About Mathematics

1. The solutions of $0 > ax^2 + bx + c$ are the x-coordinates of the solutions of $y > ax^2 + bx + c$ that are also on the x-axis.
2. The minimum value of $x^2 + 2$ is $(0, 2)$. The range is $y \geq 2$ and never intersects $y = -2$.

Developing Skills

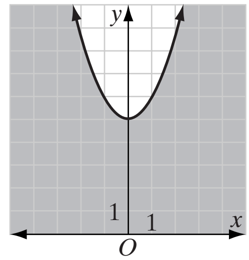
3. $(-1, 3)$ and $(3, 3)$ **4.** $(-4, 4)$ and $(2, -2)$
5. $(1, 5)$ and $(3, 1)$ **6.** $(1, 4)$ and $(4, 1)$
7. $(2, 2)$ and $(4, 4)$ **8.** $(1, 2)$ and $(-2, -1)$
9. $(-1, 2)$ and $(4, 7)$ **10.** $(-1, -1)$ and $(1, 3)$
11. $(2, 2)$ and $(3, 1)$ **12.** $(1, 4)$ and $(4, 7)$
13. $(-0.5, 2.5)$ and $(0, 3)$ **14.** $(0.5, 2.5)$ and $(3, 5)$
15. $(0.5, 2.25)$ and $(4.5, 6.25)$ **16.** $(-8, -6)$ and $(3, 5)$
17. $(3, 6)$ **18.** $(0, 0)$ and $(5, 15)$
19. $(-1, -3)$ **20.** $(-1, 2)$ and $(-2, 3)$
21. $(2, -6)$ and $(8, 6)$ **22.** $(0.5, 2)$ and $(2, 5)$
23. $(0.5, 3.25)$ and $(4, 5)$ **24.** $(0.5, 4.5)$ and $(3, 7)$
25. $(-2.5, -3.25)$ and $(2, -1)$
26. $(-2.5, 30)$ and $(3.5, 18)$
27. no real common solutions
28. $(-\sqrt{2}, 6 - 4\sqrt{2})$ and $(\sqrt{2}, 6 + 4\sqrt{2})$
29. $(-\sqrt{3}, 7 - \sqrt{3})$ and $(\sqrt{3}, 7 + \sqrt{3})$
30. $(1 - \sqrt{2}, 5 - 2\sqrt{2})$ and $(1 + \sqrt{2}, 5 + 2\sqrt{2})$
31. $(2 - \sqrt{5}, 28 - 11\sqrt{5})$ and $(2 + \sqrt{5}, 28 + 11\sqrt{5})$
32. $(1.4, 15)$ and $(0, 1)$
33. $(4 + 2i, 4 - 2i)$ and $(4 - 2i, 4 + 2i)$
34. $(4, 2)$ and $(2, -4)$
35. $\left(-\frac{4\sqrt{5}}{5}, -\frac{8\sqrt{5}}{5}\right)$ and $\left(\frac{4\sqrt{5}}{5}, \frac{8\sqrt{5}}{5}\right)$
36. $y = -2x, y = x^2 - 2$
37. $y = -x + 5, y = -(x - 2)^2 + 5$
38. (1) **39.** (4) **40.** (2)
41. (3) **42.** (6) **43.** (5)

44. a.



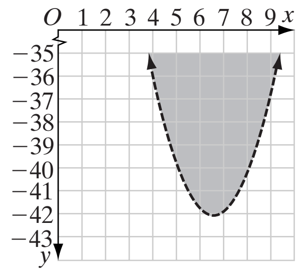
b. no

45. a.



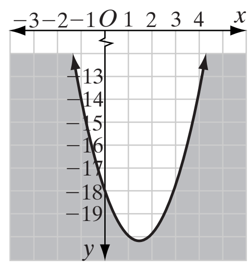
b. yes

46. a.



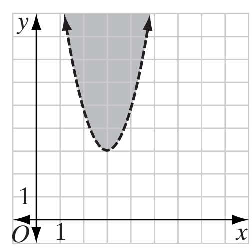
b. no

47. a.



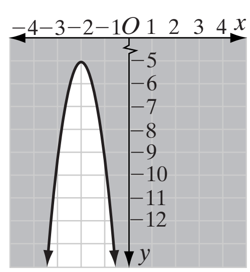
b. yes

48. a.

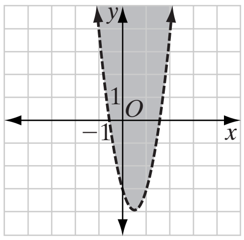


b. no

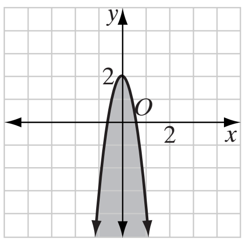
49. a.



b. yes

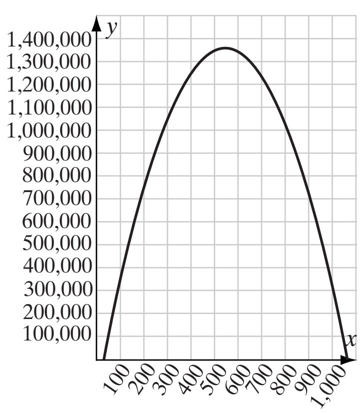
50. a. 

b. yes

51. a. 

b. no

Applying Skills

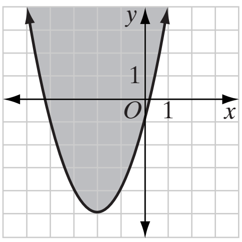
52. Width = 6 ft, length = 8 ft
53. 6 m, 7 m
54. 4 ft by 8 ft
55. a. $(x - 4)^2 + (y - 2)^2 = 20$
b. Graph
c. (0, 4) and (6, −2)
56. a. Graph
b. Yes. The graphs intersect.
c. $\left(3 - \sqrt{6}, 8 - 2\sqrt{6}\right)$ and $\left(3 + \sqrt{6}, 8 + 2\sqrt{6}\right)$
57. a. Graph
b. No. The graphs do not intersect.
c. $x = 1 \pm 2i$, $y = 2 \pm 4i$
58. (−2, −3), (0, 1), (2, 5)
59. $0.4 < t < 2.8$
60. a. $x - 4$
b. $V = 2(x - 4)^2$
c. $x > 12$
61. a. 

b. more than 20 and less than 1,060

Review Exercises *(pages 241–243)*

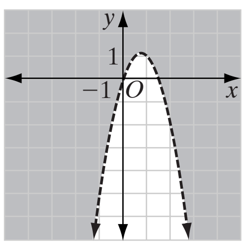
1. i **2.** $4i$ **3.** $3i$
4. $2i\sqrt{3}$ **5.** $7i$ **6.** $7i\sqrt{2}$
7. $-4i$ **8.** $\frac{4\sqrt{6}}{3}$ **9.** $7 - i$
10. $0 + 3i$ **11.** $4 + 0i$ **12.** $0 + 0i$
13. $-4 - 2i$ **14.** $10 - 4i$ **15.** $0 - 10i$
16. $0 + 6i$ **17.** $17 + 11i$ **18.** $12 + 16i$
19. $80 - 18i$ **20.** $6 + 12i$ **21.** $2 - \frac{3}{2}i$
22. $1 + 0i$ **23.** $1 - i$ **24.** $2 + 0i$
25. $\frac{-5}{13} + \frac{12}{13}i$ **26.** $\frac{2}{5} - \frac{1}{5}i$ **27.** $-15 + 8i$
28. $5 - 12i$ **29.** $\frac{7 \pm 3\sqrt{5}}{2}$ **30.** $4, -3$
31. $-2 \pm i$ **32.** $3 \pm \sqrt{19}$ **33.** $3 \pm i$
34. $-7.5, 4$ **35.** $1 \pm i$ **36.** $0.5, -2$
37. $\frac{1 \pm \sqrt{5}}{2}$ **38.** $\frac{1 \pm \sqrt{3}}{2}$ **39.** $\frac{1 \pm \sqrt{6}}{5}$
40. $\frac{3}{2} \pm i$ **41.** $2, \pm 1$ **42.** $\pm 1, \pm 2$
43. $-10 \pm 20i$ **44.** $-\frac{2}{3}, \frac{5}{6}$ **45.** $0, 3 \pm \sqrt{5}$
46. $\pm 1, \pm 3$ **47.** $-4, \pm\frac{\sqrt{3}}{3}$ **48.** $-6, -\frac{5}{3}, 1$
49. translated 1 left, 1 up
50. translated $\frac{3}{2}$ left, $7\frac{3}{4}$ up
51. scaled by 4, translated $\frac{3}{4}$ right, $\frac{3}{4}$ up
52. reflected x-axis, scaled by 2, translated $\frac{5}{4}$ right, $6\frac{7}{8}$ down
53. a. real, rational, unequal
b. real, irrational, unequal
c. No. The parabola crosses the x-axis in two distinct real points.
54. Yes, the discriminant is positive.
55. (2, 0) and (−3, 5) **56.** (1, 0) and (4, 3)
57. (0, 0) and (5, 10) **58.** (3, 4) and (4, 3)
59. (6, 0) and (3, 3) **60.** (−1, 4) and (1, 2)
61. (−2, 6) and (3, 1) **62.** (−2, −4) and (4, 8)
63. (−1, −4) and (3, 0) **64.** (3, 24) and (6, 12)
65. $\left(\frac{7 - \sqrt{29}}{2}, 2 - \sqrt{29}\right)$ and $\left(\frac{7 + \sqrt{29}}{2}, 2 + \sqrt{29}\right)$
66. $\left(\frac{-1 - \sqrt{5}}{2}, \frac{1 - \sqrt{5}}{2}\right)$ and $\left(\frac{-1 + \sqrt{5}}{2}, \frac{1 + \sqrt{5}}{2}\right)$
67. $(-0.5, 1.75)$ and $\left(\frac{5}{3}, 5\right)$
68. $(1 - i, i)$ and $(1 + i, -i)$
69. $(-2 - i, -5 - 2i)$ and $(-2 + i, -5 + 2i)$
70. (2, 1) and (−1, −2)
71. $x^2 - 2x - 15 = 0$
72. $x^2 + \frac{7}{2}x - 2 = 0$ or $2x^2 + 7x - 4 = 0$
73. $x^2 - 5$ **74.** $x^2 - 10x + 7 = 0$
75. $x^2 - 12x + 40 = 0$ **76.** 9
77. $-4 < b < 4$ **78.** $c \leq \frac{9}{4}$

79. a.



b. No

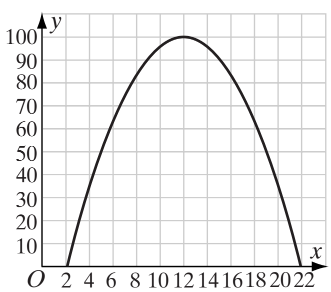
80. a.



b. Yes

81. 8.3 m by 11.7 m

82. a.



b. \$2 or \$22

c. The maximum profit is \$100 when the price is \$12.

83. The shorter side must be longer than 10 inches.

Exploration *(pages 243–244)*

1. $(x - 1)(x - 2)(x + 2)$

2. $(x^2 + x - 2)(x - 2)$

3. $\begin{array}{rrrr|l} 1 & -1 & -4 & 4 & \underline{2} \\ & 2 & 2 & -4 & \\ \hline 1 & 1 & -2 & 0 & \end{array}$

4. They are the same.

In (1)–(4), parts **a** and **b**, answers will vary depending on the choice of root used.

1. c. Yes

d. Roots = 1, 2, 3; factors = $(x - 1), (x - 2), (x - 3)$

2. c. Yes

d. Roots = −3, −1, 2; factors = $(x + 3), (x + 1), (x - 2)$

3. c. Yes

d. Roots = 1, 3; factors = $(x - 1), (x - 1), (x - 3)$

4. c. Yes

d. Roots = 1, 2; factors = $(x - 1), (x - 1), (x - 2)$

Cumulative Review *(pages 244–246)*

Part I

1. 3 **2.** 2 **3.** 2
4. 1 **5.** 2 **6.** 3
7. 3 **8.** 2 **9.** 1
10. 1

Part II

11. $\frac{2 - i}{3 - i} \cdot \frac{3 + i}{3 + i} = \frac{7}{10} - \frac{1}{10}i$

12. $6 \cdot \frac{x - 3}{2} = \frac{2x + 1}{3} \cdot 6$

$3x - 9 = 4x + 2$

$x = -11$

Part III

13. $\frac{3 + \sqrt{5}}{3 - \sqrt{5}} \cdot \frac{3 + \sqrt{5}}{3 + \sqrt{5}} = \frac{14 + 6\sqrt{5}}{4} = \frac{7 + 3\sqrt{5}}{2}$

14. $3 - |2x| = 0$

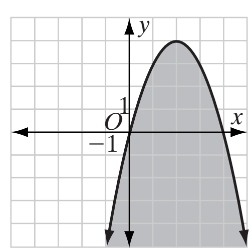
$|2x| = 3$

$2x = -3 \quad | \quad 2x = 3$

$x = -\frac{3}{2} \quad | \quad x = \frac{3}{2}$

Part IV

15. a–b.



16. a. $f \circ g(-3) = 2((-3)^2) + 4 = 22$

b. $h(x) = 2x^2 + 4$

Chapter 6. Sequences and Series

6-1 Sequences *(pages 250–252)*

Writing About Mathematics

1. Randi. Unless an upper limit is defined, the sequence is infinite.
2. **a.** Yes. $a_{n+1} = 3(n + 1) - 1 = 3n + 2 = (3n - 1) + 3 = a_n + 3$.
   b. Yes. $a_n = 2^n$ for any integer n, including $n + 1$.

Developing Skills

3. 1, 2, 3, 4, 5
4. 6, 7, 8, 9, 10
5. 2, 4, 6, 8, 10
6. $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}$
7. $\frac{1}{2}, 1, \frac{3}{2}, 2, \frac{5}{2}$
8. 19, 18, 17, 16, 15
9. 3, 9, 27, 81, 243
10. 1, 4, 9, 16, 25
11. 5, 7, 9, 11, 13
12. 1, 3, 5, 7, 9
13. $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}$
14. $3, 2, \frac{5}{3}, \frac{3}{2}, \frac{7}{5}$
15. $-1, -2, -3, -4, -5$
16. $9, 6, 3, 0, -3$
17. $\frac{4}{3}, \frac{8}{3}, 4, \frac{16}{3}, \frac{20}{3}$
18. $\frac{1}{2} + i, 1 + i, \frac{3}{2} + i, 2 + i, \frac{5}{2} + i$
19. **a.** $a_n = 2n$ **b.** 18
20. **a.** $a_n = 3n$ **b.** 27
21. **a.** $a_n = 3n - 2$ **b.** 25
22. **a.** $a_n = 3^n$ **b.** $3^9 = 19{,}683$
23. **a.** $a_n = \frac{24}{2^n}$ **b.** $\frac{24}{2^9} \approx 0.047$
24. **a.** $a_n = 2n + 5$ **b.** 23
25. **a.** $a_n = 12i - 2ni$ **b.** $-6i$
26. **a.** $a_n = \frac{1}{n + 1}$ **b.** $\frac{1}{10}$
27. **a.** $a_n = \frac{n}{n + 1}$ **b.** $\frac{9}{10}$
28. **a.** $a_n = n^2 + 1$ **b.** 82
29. **a.** $a_n = n \cdot (-1)^{n+1}$ **b.** 9
30. **a.** $a_n = \sqrt{n}$ **b.** 3
31. 5, 6, 7, 8, 9
32. 1, 3, 9, 27, 81
33. 1, 3, 7, 15, 31
34. $-2, 4, -8, 16, -32$
35. 20, 16, 12, 8, 4
36. 4, 5, 7, 10, 14
37. $108, 36, 12, 4, \frac{4}{3}$
38. 4, 10, 25, 62.5, 156.25
39. $\frac{1}{2}, 2, \frac{1}{2}, 2, \frac{1}{2}$

Applying Skills

40. **a.** 30, 35, 40, 45, 50, 55, 60
b. $a_{n+1} = a_n + 5, a_1 = 30$
41. **a.** 4, 6, 8, 10, 12, 14, 16
b. $a_{n+1} = a_n + 2, a_1 = 4$
42. **a.** 180, 178, 176, 174, 172, 170, 168, 166, 164
b. $a_{n+1} = a_n - 2, a_1 = 180$
43. **a.** Jan 1, Jan 8, Jan 15, Jan 22, Jan 29
b. $a_{n+1} = a_n + 7, a_1 = 1$
44. **a.** \$400, \$440, \$484, \$532.40, \$585.64, \$644.20, \$708.62
b. $a_{n+1} = 1.1a_n, a_1 = \$400$
45. 1, 1, 2, 3, 5, 8, 13, 21, 34, 55

Hands-On Activity

1. **a.** 3 **b.** 7 **c.** 15 **d.** $2^n - 1$
2. $a_{n+1} = 2a_n + 1, a_1 = 3$

6-2 Arithmetic Sequences *(pages 256–257)*

Writing About Mathematics

1. Virginia's solution works, but it is not a better method. As the value of n increases, her method becomes more and more time-consuming.
2. No. Pedro's method yields an arithmetic sequence of *six* terms, not five.

Developing Skills

3. Yes, $d = 3$
4. Yes, $d = 2i$
5. No
6. Yes, $d = -5$
7. No
8. Yes, $d = 0.25$
9. **a.** $d = 3$ **b.** 24
10. **a.** $d = 5$ **b.** 57
11. **a.** $d = -2$ **b.** 0
12. **a.** $d = \frac{1}{2}$ **b.** $\frac{7}{2}$
13. **a.** $d = -2$ **b.** -19
14. **a.** $d = 0.1$ **b.** 4.0
15. 12, 18, 24, 30, 36, 42
16. 120, 115, 110, 105, 100, 95, 90, 85, 80
17. 6, 9, 12, 15
18. $\frac{7}{3}, \frac{11}{3}$
19. $a_{n+1} = a_n - 3$

Applying Skills

20. \$6,000, \$5,600, \$5,200, \$4,800, . . .
The amount owed each month has a common difference, -400.
21. Week 9
a. $60 = 20 + (n - 1)5$
b. 20, 25, 30, . . . , 60
c. Using a formula is more efficient for long sequences.
22. **a.** Choose any linear function and set up a chart, showing that for each integer value of x, y increases by a fixed amount.
b. $a_1 = b, d = m$
23. **a.** 40
b. 154

6-3 Sigma Notation *(pages 260–261)*

Writing About Mathematics

1. Yes. The first and last terms of the series have been decreased by 2, and then re-increased by 2 in the expression evaluated by sigma.

2. $\frac{1}{k}$ is undefined for $k = 0$.

Developing Skills

3. a. $3 + 6 + 9 + 12 + 15 + 18 + 21 + 24 + 27 + 30$
   b. 165
4. a. $0 + 2 + 4 + 6 + 8$
   b. 20
5. a. $1 + 4 + 9 + 16$
   b. 30
6. a. $1 + 8 + 27 + 64 + 125 + 216$
   b. 441
7. a. $95 + 90 + 85 + 80 + 75 + 70 + 65 + 60 + 55 + 50$
   b. 725
8. a. $12 + 15 + 18 + 21 + 24 + 27$
   b. 117
9. a. $(4 + 2i) + (9 + 2i) + (16 + 2i) + (25 + 2i)$
   b. $54 + 8i$
10. a. $-1 + 2 - 3 + 4 - 5 + 6 - 7 + 8 - 9 + 10$
    b. 5
11. a. $14 + 17 + 20 + 23 + 26 + 29 + 32 + 35 + 38 + 41 + 44$
    b. 319
12. a. $-i - 2i - 3i - 4i - 5i - 6i - 7i - 8i - 9i - 10i$
    b. $-55i$
13. a. $-7 - 11 - 15$
    b. -33
14. a. $0 + 4 - 64 + 1{,}296 - 32{,}768 + 1{,}000{,}000$
    b. 968,468

15. $\sum_{n=1}^{7}(2n + 1)$
16. $\sum_{n=1}^{8}(5n - 4)$
17. $\sum_{n=1}^{5}n^n$
18. $\sum_{n=0}^{19}(100 - 5n)$
19. $\sum_{n=1}^{10}3n$
20. $\sum_{n=1}^{5}\frac{1}{2^{n-1}}$
21. $\sum_{n=1}^{9}\frac{n}{n + 1}$
22. $\sum_{n=1}^{5}\frac{1}{n!}$
23. $\sum_{n=1}^{5}(-1)^n\left(\frac{n}{3^n}\right)$
24. $\sum_{n=1}^{6}\frac{1}{n(n + 1)}$
25. $\sum_{n=1}^{\infty}n^2$
26. $\sum_{1}^{\infty}\left(\frac{n}{3^n}\right)$

Applying Skills

27. $ka_1 + ka_2 + \cdots = k(a_1 + a_2 + \cdots)$
28. $(a_1 + b_1) + (a_2 + b_2) + (a_3 + b_3) + \cdots = (a_1 + a_2 + a_3 + \cdots) + (b_1 + b_2 + b_3 + \cdots)$
29. a. $a_n = 20 + (n - 1) \cdot 3 = 3n + 17$
    b. $\sum_{n=1}^{35}[20 + 3(n - 1)] = \sum_{n=1}^{35}(3n + 17)$
30. a. $a_n = 45 + (n - 1) \cdot 15 = 15n + 30$
    b. $\sum_{n=1}^{5}[45 + 15(n - 1)] = \sum_{n=1}^{5}(15n + 30)$
31. (1) 54.50
    (2) 12.74
    (3) 0.67

6-4 Arithmetic Series *(pages 264–265)*

Writing About Mathematics

1. Yes. $1{,}200 = \frac{n}{2}(80)$, yielding $n = 30$. This is true for *any* arithmetic series with 30 terms such that $a_1 + a_n = 80$.
2. No. The difference between terms is not constant.

Developing Skills

3. 42
4. 210
5. -45
6. $60i$
7. 7
8. $120\sqrt{2}$
9. a. $\sum_{n=1}^{10}[3 + 4(n - 1)] = \sum_{n=1}^{10}(4n - 1)$
   b. 210
10. a. $\sum_{n=1}^{6}[24 - 4.8(n - 1)] = \sum_{n=1}^{6}(-4.8n + 28.8)$
    b. 72
11. a. $\sum_{n=1}^{5}[24 - 6(n - 1)] = \sum_{n=1}^{5}(-6n + 30)$
    b. 60
12. a. $\sum_{n=1}^{14}[10 + 2(n - 1)] = \sum_{n=1}^{14}(2n + 8)$
    b. 322
13. a. $\sum_{n=1}^{15}[2 + \frac{1}{2}(n - 1)] = \sum_{n=1}^{15}(\frac{1}{2}n + \frac{3}{2})$
    b. $\frac{165}{2}$
14. a. $\sum_{n=1}^{10}-2(n - 1) = \sum_{n=1}^{10}-2n + 2$
    b. -90
15. a. $\sum_{n=1}^{12}\frac{1}{3}n$ b. 26
16. a. $\sum_{n=1}^{20}[100 - 5(n - 1)] = \sum_{n=1}^{20}[-5n + 105]$
    b. 1,050
17. a. $\sum_{n=1}^{10}[27.5 + 2.5(n - 1)] = \sum_{n=1}^{10}(2.5n + 25)$
    b. 387.5
18. a. $\sum_{n=1}^{12}[7 + 2(n - 1)] = \sum_{n=1}^{12}(2n + 5)$
    b. 216
19. a. $2 + 4 + 6 + \cdots + 20$
    b. 110

20. a. $5 + 6 + 7 + \cdots + 11$ **b.** 56
21. a. $20 + 18 + 16 + \cdots + 2$ **b.** 110
22. a. $100 + 95 + 90 + \cdots + 5$ **b.** 1,050
23. a. $-2 - 4 - 6 - \cdots - 50$ **b.** -650
24. a. $1 + 3 + 5 + \cdots + 19$ **b.** 100

Applying Skills

25. 45
26. a. 11 days **b.** 22 miles
27. $120
28. 2,135 seats
29. 375 minutes
30. $129,000
31. $10,350

6-5 Geometric Sequences *(pages 269–270)*

Writing About Mathematics

1. Answers will vary. This method works fine for small sequences, but is inefficient for large values of n.
2. Yes. There are three geometric means between 8 and 32.

Developing Skills

3. Yes, $r = 2$
4. Yes, $r = 5$
5. No, arithmetic
6. Yes, $r = 4$
7. Yes, $r = -3$
8. Yes, $r = \frac{1}{3}$
9. Yes, $r = \frac{1}{3}$
10. No, arithmetic
11. Yes, $r = -10$
12. Yes, $r = 0.1$
13. Yes, $r = -2$
14. Yes, $r = a$
15. 1; 6; 36; 216; 1,296
16. $40, 20, 10, 5, \frac{5}{2}$
17. 2, 6, 18, 54, 162
18. $\frac{1}{4}, \frac{-1}{2}, 1, -2, 4$
19. $1, \sqrt{2}, 2, 2\sqrt{2}, 4$
20. 10, 30, 90, 270, 810
21. $-1, 4, -16, 64, -256$
22. 100, 10, 1, 0.1, 0.01 or 100, -10, 1, -0.1, 0.01
23. 1, 4, 16, 64, 256 or 1, -4, 16, -64, 256
24. $1, \sqrt{2}, 2, 2\sqrt{2}, 4$ or $1, -\sqrt{2}, 2, -2\sqrt{2}, 4$
25. $1, -2, 4, -8, 16$
26. 81, 27, 9, 3, 1 or 81, -27, 9, -3, 1
27. 128
28. 0.00032 or $\left(\frac{1}{5}\right)^5$
29. 256
30. 2,187
31. $\pm 256\sqrt{2}$
32. $\frac{1}{3}$
33. 15, 37.5
34. $4, \frac{16}{3}, \frac{64}{9}$ or $-4, \frac{16}{3}, -\frac{64}{9}$
35. $24\sqrt{2}, 144, 432\sqrt{2}$ or $-24\sqrt{2}, 144, -432\sqrt{2}$

Applying Skills

36. $1,000, $1,060, $1,123.60, $1,191.02, $1,262.48, $1,338.23, $1,418.52, $1,503.63, $1,593.85, $1,689.48
37. $3,150, $3,307.50, $3,472.88, $3,646.52
38. 5,000; 4,900; 4,802; 4,706; 4,611; 4,520; 4,429; 4,341
39. 55, 61, 67, 73, 81
40. $16,000, $12,800, $10,240, $8,192
41. $42,500, $36,125, $30,706.25, $26,100.31, $22,185.27, $18,857.48

6-6 Geometric Series *(pages 272–273)*

Writing About Mathematics

1. Yes. $a_n = a_1r^{n-1}$, so $a_nr = a_1r^n$.
2. Probably not. This method becomes especially cumbersome with large values of n.

Developing Skills

3. 4,095
4. 354,292
5. 536,870,911.5
6. 1,111,110
7. 409.5
8. $\frac{3^{10} - 1}{2 \cdot 3^9} = \frac{59{,}048}{39{,}366} \approx 1.49997$
9. 1,275
10. 2,441,406
11. $\frac{63}{64}$
12. 6,554
13. 39,364 or 19,684
14. 1,023
15. a. $3 + 6 + 12 + 24 + 48 + 96$
b. 189
16. a. $1 + \frac{1}{3} + \frac{1}{9} + \cdots + \frac{1}{243}$
b. $\frac{364}{243} \approx 1.4979$
17. a. $10 + 5 + \frac{5}{2} + \cdots + \frac{5}{6}$
b. $\frac{315}{16} = 19.6875$
18. a. $-6 - 24 - 96 - 384 - 1{,}536 - 6{,}144 - 24{,}576 - 98{,}304 - 393{,}216$
b. 524,286
19. a. $1 - 2 + 4 - 8 + 16 - 32$
b. -21
20. a. $1 + \frac{2}{3} + \frac{4}{9} + \cdots + \frac{32}{243}$
b. $\frac{665}{243} \approx 2.7366$
21. a. $100 + 50 + 25 + \cdots + \frac{100}{64}$
b. $\frac{3{,}175}{16} = 198.4375$
22. a. $-81 + 27 + -9 + \cdots + \frac{-1}{9}$
b. $-\frac{547}{9} = -60.\overline{7}$
23. $13 + 13\sqrt{3}$
24. $\dfrac{1 - 625^{\frac{8}{7}}}{1 - 625^{\frac{1}{7}}} \approx 1{,}038.66$
25. 1,023
26. a. $400(1.05)^1 = \$420$
b. Yes, $r = 1.05$
c. $2,856.80
27. $17\frac{8}{9}$ feet
28. 20 days

6-7 Infinite Series *(page 278)*

Writing About Mathematics

1. $S = \frac{a}{1 - r} = \frac{1}{1 - \frac{1}{c}} = \frac{c}{c - 1}$

2. No. The calculator's value of *e* is only an approximation. *e* is an irrational number.

Developing Skills

3. a. $1 + \sum_{n=1}^{\infty}\left(\frac{1}{3}\right)^n$
   b. Finite limit: $\frac{3}{2}$
4. a. $2 + \sum_{n=1}^{\infty}2\left(\frac{1}{4}\right)^n$
   b. Finite limit: $\frac{8}{3}$
5. a. $\sum_{n=1}^{\infty}2n$
   b. Increases without limit
6. a. $5 + \sum_{n=1}^{\infty}5\left(\frac{1}{5}\right)^n$
   b. Finite limit: $\frac{25}{4}$
7. a. $5 + \sum_{n=1}^{\infty}(5 - 4n)$
   b. Decreases without limit
8. a. $6 + \sum_{n=1}^{\infty}6\left(\frac{1}{2}\right)^n$
   b. Finite limit: 12
9. a. $\sum_{n=1}^{\infty}\frac{1}{(n + 1)!}$
   b. Finite limit: $(e - 2)$
10. a. $\sum_{n=1}^{\infty}\frac{n(n + 1)}{2}$
    b. Increases without limit
11. $\frac{10}{9}$
12. $\frac{1}{3}$
13. $\frac{4}{9}$
14. $\frac{12}{99}$
15. $\frac{24}{99}$
16. $\frac{126}{999}$
17. $1 \leq n < 25$

Review Exercises *(pages 280–281)*

1. a. $a_{n+1} = a_n + 4, a_1 = 1$
   b. Arithmetic
   c. $a_n = 1 + 4(n - 1) = 4n - 3$
   d. 37
2. a. $a_{n+1} = \frac{a_n}{3}, a_1 = 3$
   b. Geometric
   c. $a_n = 3\left(\frac{1}{3}\right)^{n-1}$
   d. $\frac{1}{6{,}561}$
3. a. $a_{n+1} = a_n - 1, a_1 = 12$
   b. Arithmetic
   c. $a_n = 12 - 1(n - 1) = -n + 14$
   d. 3
4. a. $a_{n+1} = a_n + n + 1, a_1 = 1$
   b. Neither
   d. 55
5. a. $a_{n+1} = a_n + i(2^n), a_1 = i$
   b. Neither
   d. $1{,}023i$
6. a. $a_{n+1} = -3a_n, a_1 = 2$
   b. Geometric
   c. $a_n = 2(-3)^{n-1}$
   d. −39,366
7. $\sum_{n=1}^{8}\frac{(n)(n + 1)}{2}$
8. $\sum_{n=0}^{6}(3n + 2)$
9. $\sum_{n=1}^{6}(2n + 2)$
10. $\sum_{n=1}^{6}(2n - 1)^2$
11. $\sum_{n=1}^{7}(-1)^{n-1} \cdot n$
12. $\sum_{n=1}^{\infty}\frac{1}{2^n}$
13. 6, 11, 16, 21, 26
14. 151
15. 71
16. −20
17. $2, 5, \frac{25}{2}, \frac{125}{4}, \frac{625}{8}$
18. $\frac{5^9}{2^8} \approx 7{,}629.3945$
19. 2,048
20. $a_{n+1} = a_n + (6 + 2n), a_1 = 12$
21. 1, 7, 31, 127, 511
22. 6, 11, 16, 21, 26, 31
23. 5, 25, 125 or −5, 25, −125
24. 12
25. $\frac{\sqrt{2}}{2}$
26. $3 + 9 + 27 + 81 = 120$
27. $12 + 9 + 6 + 3 + 0 - 3 - 6 = 21$
28. 60.26
29. a. $\sum_{n=1}^{\infty}3 \cdot \left(\frac{1}{2}\right)^{n-1}$
    b. Finite limit: 6
30. a. 8 cans
    b. 108 cans
31. a. \$24,500
    b. \$222,500
32. a. \$52,637.27
    b. \$368,569.05
33. a. $a_n = a_{n-1} + n - 1$
    b. 0, 1, 3, 6, 10, 15, 21, 28, 36, 45
    c. $a_n = \frac{n(n - 1)}{2}$

Exploration *(page 282)*

In 1–4, part **a**, answers will be graphs.

1. b. Diverges **2. b.** Oscillates
3. b. Converges **4. b.** Converges

Cumulative Review *(pages 283–285)*

Part I

1. 4 **2.** 2 **3.** 1
4. 1 **5.** 3 **6.** 3
7. 1 **8.** 3 **9.** 2
10. 4

Part II

11. *Answer:* $\frac{9}{4}$

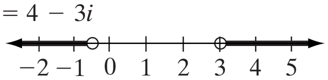
$$x^2 + 3x + \frac{9}{4} = \frac{9}{4}$$
$$\left(x + \frac{3}{2}\right)^2 = \frac{9}{4}$$

12. $7 - \sqrt{x+2} = 4$
$$-\sqrt{x+2} = -3$$
$$x + 2 = 9$$
$$x = 7$$

Part III

13. $\frac{(2+i)^2}{i}$
$= \frac{3+4i}{i}$
$= \frac{3+4i}{i} \cdot \frac{i}{i}$
$= \frac{3i-4}{-1}$
$= 4 - 3i$

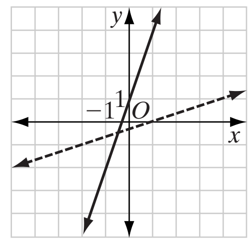
14.



Part IV

15. a. $y = \frac{x-1}{3}$

b.



c. $\left(-\frac{1}{2}, -\frac{1}{2}\right)$

Set both equations equal to each other:

$$\frac{x-1}{3} = 3x + 1$$
$$x - 1 = 9x + 3$$
$$-8x = 4$$
$$x = -\frac{1}{2}$$

Substitute this value of x into either equation to find the y-coordinate.

16. a. $a_{n+1} = 10a_n, a_1 = 3$

b. $\sum_{n=1}^{5} 3(10)^{n-1}$

c. 33,333

Chapter 7. Exponential Functions

7-1 Laws of Exponents *(pages 288–289)*

Writing About Mathematics

1. No, they do not share a common base or common exponent. $(2)^3(5)^2 = (8)(25) = 200$. $10^5 = 10{,}000$.
2. Yes, this is true via the commutative property.

Developing Skills

3. x^7 **4.** y^6 **5.** x^4
6. y^3 **7.** x^{10} **8.** $8y^{12}$
9. 10^6 **10.** -2^8 **11.** x^6y^3
12. x^2y^7 **13.** $-9x^6$ **14.** $9x^6$
15. x^5y **16.** x **17.** x^8y^{10}
18. $64x^{10}$ **19.** 16 **20.** x^5y^5
21. xy^2 **22.** x^2y^2 **23.** $\frac{y^7z^2}{x^2}$
24. $32a^5b$ **25.** $4abc^4$ **26.** b

Applying Skills

27. 9 **28.** 3
29. $y = \frac{x}{3}$ **30.** $x = 25y^2$
31. \$608.33 **32.** \$3,909.35
33. 15 years

7-2 Zero and Negative Exponents *(pages 292–293)*

Writing About Mathematics

1. No. $a^0 + a^0 = 2a^0 = 2(1) = 2$
2. Yes. $a^0 + a^0 = 1 + 1 = 2$ and $2a^0 = 2$.

Developing Skills

3. $\frac{1}{5}$ **4.** $\frac{1}{16}$ **5.** $\frac{1}{36}$
6. 2 **7.** 125 **8.** $\frac{3}{2}$

9. 16 | 10. $\frac{1}{400}$ | 11. 1
12. 1 | 13. 1 | 14. -1
15. 1 | 16. 4 | 17. -2
18. 1 | 19. 1 | 20. $\frac{1}{4}$
21. 1 | 22. 3 | 23. 1
24. $\frac{730}{243}$ | 25. $-\frac{1}{1,259,712}$ | 26. $\frac{1}{(-6)^{28}} = \frac{1}{6^{28}}$

27. $\frac{1}{4}$
28. $\frac{10^9 + 1}{10^6} = 1,000.000001$

29. $\left(\frac{4}{3}\right)^{15} \approx 74.8309$
30. $\sqrt{3}$

31. $\frac{3}{64}$ | 32. $\frac{1}{32}$ | 33. $\frac{1}{\sqrt{2}}$ or $\frac{\sqrt{2}}{2}$
34. 56 | 35. $\frac{1}{x^4}$ | 36. $\frac{1}{a^6}$
37. $\frac{1}{y^5}$ | 38. $\frac{2}{x^2}$ | 39. $\frac{7}{a^4}$
40. $\frac{-5}{y^8}$ | 41. $\frac{1}{4x^2}$ | 42. $\frac{1}{81a^4}$
43. $\frac{1}{2}$ | 44. $-\frac{1}{4x^2}$ | 45. $-\frac{1}{81a^4}$
46. $\frac{1}{4x^2}$ | 47. x^3 | 48. y^7
49. $3a^5$ | 50. $6a^4$ | 51. $9a^3x^2$
52. $-\frac{1}{x^2}$ | 53. $\frac{a}{4}$ | 54. $\frac{2}{y^2}$
55. $\frac{z^2}{xy^5}$ | 56. $\frac{b^{15}c^{12}}{a^{15}}$ | 57. $\frac{8m^9}{27n^6}$
58. $\frac{1}{2ab^4x^3y^4}$ | 59. $\frac{x^9}{y^2}$ | 60. $9b$
61. $\frac{-32b^4}{xa^2}$ | 62. $\frac{uv^3}{7}$ | 63. $\frac{x^4 + 1}{x^5}$
64. y^{-2} | 65. ab^{-2} | 66. y^{-4}
67. $6b^{-1}$ | 68. a^3 | 69. $3x^{-4}$
70. $2a^{-3}$ | 71. $4x^5$ | 72. $3a$
73. $5xy^{-10}$ | 74. $5x^{-3}y^{-3}$ | 75. $25a^6b^{-4}$
76. $2\frac{1}{8}$ | 77. $\frac{4}{9}$ | 78. $a = 8$, $b = 3$

79. $3 \times \frac{1}{100}$

7-3 Fractional Exponents *(pages 296–298)*

Writing About Mathematics

1. $(\sqrt[n]{a})^0 = a^{\frac{0}{n}} = a^0 = 1$
2. $(a^{\frac{1}{2}})^{\frac{1}{2}} = a^{\frac{1}{2}\cdot\frac{1}{2}} = a^{\frac{1}{4}} = \sqrt[4]{a}$

Developing Skills

3. 2 | 4. 3 | 5. 10
6. 2 | 7. 5 | 8. 6
9. 2 | 10. 6 | 11. 2
12. 15 | 13. -40 | 14. 343
15. 32 | 16. 81 | 17. 1,000
18. 16 | 19. $\frac{1}{3}$ | 20. $\frac{1}{2}$
21. $\frac{1}{1,000}$ | 22. $\frac{1}{25}$ | 23. 9
24. 5 | 25. 49 | 26. 8

27. 64 | 28. 2 | 29. 12
30. 9 | 31. 2 | 32. 5
33. $1\frac{1}{2}$ | 34. $3\frac{1}{81}$ | 35. $\frac{24}{5}$
36. 1.196 | 37. $\frac{9}{4}$ | 38. $7^{\frac{1}{2}}$
39. $6^{\frac{1}{2}}$ | 40. $12^{\frac{1}{3}}$ | 41. $15^{\frac{1}{3}}$
42. $3^{\frac{1}{4}}$ | 43. $2^{\frac{3}{5}}$ | 44. $9^{\frac{4}{5}}$
45. $\frac{1}{5^{\frac{3}{2}}}$ | 46. $5a^{\frac{1}{2}}$ | 47. $7x$
48. $8a^{\frac{3}{2}}b^3$ | 49. $\frac{3}{2}(2)^{\frac{1}{2}}a^3b$ | 50. $\frac{3b^3}{a}$
51. $\left(\frac{3a}{4b}\right)^{\frac{1}{2}}$ | 52. $3a$ | 53. $64^{\frac{1}{4}}x^{\frac{5}{4}} = 2^{\frac{3}{2}}x^{\frac{5}{4}}$
54. $\frac{1}{(xy)^{\frac{1}{5}}z}$ | 55. $\frac{3}{2ab^2}$ | 56. $\frac{w^{\frac{15}{10}}x^2}{y^{\frac{5}{10}}} = \frac{w^{\frac{3}{2}}x^2}{y^{\frac{1}{2}}}$
57. $(a^{\frac{1}{4}} \cdot b^{\frac{7}{4}})^{\frac{1}{8}} = a^{\frac{1}{32}} \cdot b^{\frac{7}{32}}$ | 58. $\sqrt{3}$

59. $\sqrt{5}$ | 60. $\sqrt[3]{6}$
61. $\sqrt[3]{9}$ | 62. $5\sqrt{5}$
63. $12\sqrt[4]{12}$ | 64. $36\sqrt{6}$
65. $\frac{\sqrt{5}}{25}$ | 66. $x\sqrt[7]{x^6}$
67. $5x\sqrt{y}$ | 68. $5b^2\sqrt{2a}$
69. $2ab\sqrt[4]{ab^2}$ | 70. $\sqrt[7]{x^5y^6z^3}$
71. $\sqrt[3]{\frac{5a^2}{4}} = \frac{\sqrt[3]{10a^2}}{2}$ | 72. $\sqrt[5]{\frac{-32x^{10}}{y^4}} = \frac{-2x^2}{\sqrt[5]{y^4}} = \frac{-2x^2\sqrt[5]{y}}{y}$

73. $\sqrt[6]{\frac{8a^5b^3}{27c^4}} = \frac{\sqrt[6]{216a^5b^3c^2}}{3c}$

74. $\frac{64a^2}{729}$ | 75. $\frac{x^{\frac{4}{3}}y^{\frac{2}{3}}}{3^{\frac{2}{3}}x^{\frac{8}{3}}b^{\frac{4}{3}}} = \frac{y^{\frac{2}{3}}}{3^{\frac{2}{3}}x^{\frac{4}{3}}b^{\frac{4}{3}}}$
76. $\frac{2a^{\frac{5}{2}}b^{\frac{5}{2}}}{5}$ | 77. $\frac{2a^2z^{\frac{7}{3}}}{3x^3}$
78. x^3y^2 | 79. $\frac{1}{a^{\frac{1}{6}}}$
80. $\frac{11^{\frac{1}{3}}y^{\frac{1}{3}}}{2^{\frac{1}{2}}x^{\frac{5}{6}}}$ | 81. $\frac{48^{\frac{1}{5}}}{6^{\frac{1}{3}}x^{\frac{7}{15}}y^{\frac{14}{15}}} = \frac{2^{\frac{7}{15}}}{3^{\frac{2}{15}}x^{\frac{7}{15}}y^{\frac{14}{15}}}$

82. $2^{\frac{3}{2}}xy^{\frac{5}{4}}$

83. **a.**
$$(\sqrt[3]{27} \cdot \sqrt[3]{3})^2 \stackrel{?}{=} (27^{\frac{1}{3}} \cdot 3^{\frac{1}{3}})^2$$
$$(\sqrt[3]{81})^2 \stackrel{?}{=} ((3^3)^{\frac{1}{3}} \cdot 3^{\frac{1}{3}})^2$$
$$\sqrt[3]{3^8} \stackrel{?}{=} (3^{1+\frac{1}{3}})^2$$
$$3^{\frac{8}{3}} = 3^{\frac{8}{3}} \checkmark$$

b.
$$\left(\frac{\sqrt{3}}{\sqrt{9}}\right)^3 \stackrel{?}{=} \left(\frac{3^{\frac{1}{2}}}{9^{\frac{1}{2}}}\right)^3$$
$$\left(\frac{\sqrt{3}}{\sqrt{3}\sqrt{3}}\right)^3 \stackrel{?}{=} \frac{3^{\frac{3}{2}}}{(3^2)^{\frac{3}{2}}}$$
$$\left(\frac{1}{\sqrt{3}}\right)^3 \stackrel{?}{=} 3^{\frac{3}{2}-3}$$
$$\frac{1}{3^{\frac{3}{2}}} \stackrel{?}{=} \frac{1}{3^{\frac{3}{2}}} \checkmark$$

7-4 Exponential Functions and Their Graphs *(pages 302–303)*

Writing About Mathematics

1. Any non-zero number raised to the zero power is 1.

2. One raised to any power is 1, thus $y = 1$ for all values of x.

Developing Skills

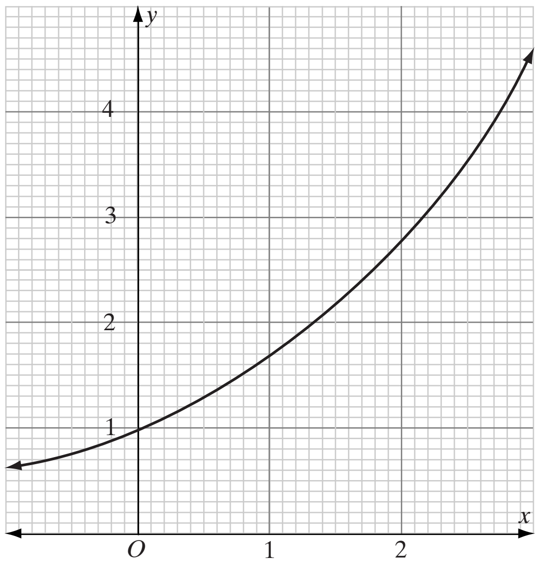
In 3–6, parts **a** and **b**, answers will be graphs.

3. c. $y = 4^{-x}$ or $y = \left(\frac{1}{4}\right)^x$

4. c. $y = 3^{-x}$ or $y = \left(\frac{1}{3}\right)^x$

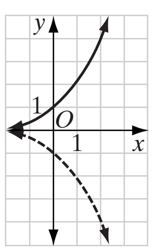
5. c. $\left(\frac{7}{2}\right)^{-x}$ or $\left(\frac{2}{7}\right)^x$ **6. c.** $\left(\frac{3}{4}\right)^{-x}$ or $\left(\frac{4}{3}\right)^x$

7. a.



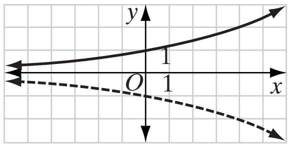
b. 3.1 **c.** 4.4

8. a–b.



c. $y = -2^x$

9. a–b.

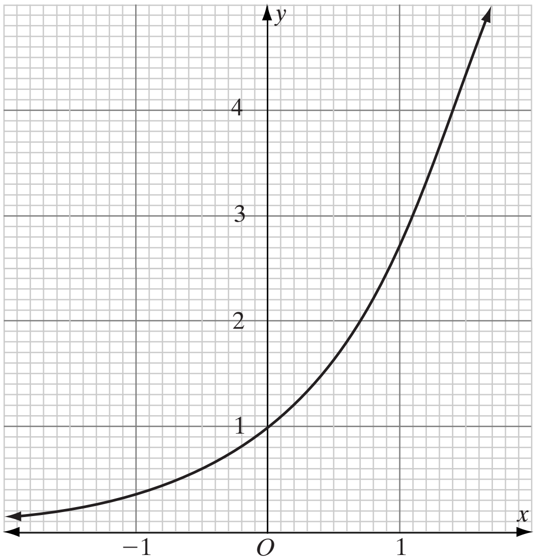


c. $y = -1.2^x$

10. a.

x	e^x
–2	0.135
–1	0.368
0	1
1	2.718
2	7.389
3	20.086

b.

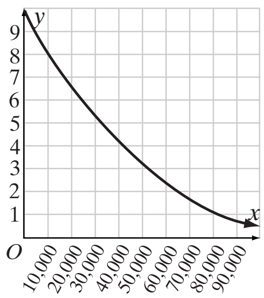


c. 1.6

Applying Skills

11. a.

b. In 2010, 338,880,723.
In 2020, 386,313,106.

12. a. 

b. In 10 years, 9.997 grams.
In 100 years, 9.971 grams.

c. Answers will vary: 79,951 years

13. a. Graph

b. 2 points: (2, 16) and (4, 64)

c. $y = 4^x$

7-5 Solving Equations Involving Exponents *(page 305)*

Writing About Mathematics

1. Yes. Squaring both sides eliminates the fractional exponent.

2. No. $a^{-2} = \frac{1}{a^2}$, but 36 does not equal its inverse.

Developing Skills

3. 64	**4.** 32	**5.** 243
6. 4	**7.** $\pm\frac{1}{3}$	**8.** 2
9. $\frac{1}{6}$	**10.** 81	**11.** 16
12. 9	**13.** 27	**14.** 72
15. 3	**16.** 5	**17.** 81
18. 0.35	**19.** 14.70	**20.** 1.24
21. 2.03	**22.** 2.20	**23.** 0.54

24.
$$x^{\frac{1}{3}-\frac{2}{3}} = 10$$
$$x^{-\frac{1}{3}} = 10$$
$$\left(x^{-\frac{1}{3}}\right)^{-3} = (10)^{-3}$$
$$x = \frac{1}{1{,}000}$$

Applying Skills

25. If the area of one face is B, then the length of one side of the cube is $\sqrt{B}$. Therefore, the volume of the cube is $(\sqrt{B})^3$ or $B^{\frac{3}{2}}$.

26. $B = V^{\frac{2}{3}}$

7-6 Solving Exponential Equations *(pages 307–308)*

Writing About Mathematics

1. $a = 0$. Anything to the zero power is 1.

2. There is no common base.

Developing Skills

3. 3^2	**4.** 3^3	**5.** 5^2
6. 7^2	**7.** 10^3	**8.** 2^5
9. $\left(\frac{1}{2}\right)^3$ or 2^{-3}	**10.** $\left(\frac{1}{6}\right)^3$ or 6^{-3}	**11.** $(0.1)^3$
12. $(0.5)^3$	**13.** $(0.9)^2$	**14.** $(0.4)^2$
15. 4	**16.** 3	**17.** −1
18. −2	**19.** 2	**20.** −2
21. $-\frac{1}{2}$	**22.** 2	**23.** 3
24. −1	**25.** $\frac{1}{2}$	**26.** −2
27. 3	**28.** 3	**29.** 6
30. 3	**31.** 2	**32.** −2
33. 1	**34.** $\frac{1}{5}$	**35.** $\frac{3}{2}$
36. 0	**37.** −3	**38.** ±2

7-7 Applications of Exponential Functions *(pages 312–313)*

Writing About Mathematics

1. 100% = 1; thus, $A = A_0(1 + 1)^n = A_0(2)^n$.

2. Daily interest earns interest on earned interest, not just the principal.

Developing Skills

3. 7.39	**4.** 4.48	**5.** 0.37
6. 2.72	**7.** 0.23	**8.** 2.72
9. 2,980.96	**10.** 168.50	**11.** 51.01
12. 344.60	**13.** $r = 100\%$	**14.** $t = 3$
15. 577.21%	**16.** −36.11%	

Applying Skills

17. a. \$1,060, \$1,123.60, \$1,191.02, \$1,262.48, \$1,338.23

b. \$1,061.68, \$1,127.16, \$1,196.68, \$1,270.49, \$1,348.85

c. Sue

d. Joe = 6%, Sue = 6.168%

18. a. \$10,129.08

b. \$10,272.17

19. \$1,508,661.82	**20.** 17.22 g
21. 31,529	**22.** \$369,452.80
23. 30.23 g	**24.** 4,000

25. a. $A = A_0e^{rt}$; medicine decreases continually.

b. Continuous = 69.99 mg
Periodic = 66.16 mg

Review Exercises *(pages 315–316)*

1. 1	**2.** $\frac{1}{2}$	**3.** $\frac{1}{2}$
4. 5	**5.** 64	**6.** 500
7. 36	**8.** $\frac{1}{36}$	**9.** 3
10. $\frac{1}{10}$	**11.** 25	**12.** $\frac{1}{64}$
13. 2	**14.** $\frac{1}{28}$	**15.** $16\frac{1}{16}$

16. 10,000 **17.** $59{,}049x^2$ **18.** $\frac{c^6}{a^6}$ or $\left(\frac{c}{a}\right)^6$

19. $\frac{1}{x^{\frac{3}{5}}b^{\frac{2}{5}}}$ **20.** $\frac{6yz^{\frac{1}{2}}}{x}$ **21.** $\frac{3}{5x^{\frac{1}{2}}}$

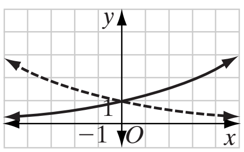
22. $\frac{3}{y^{\frac{2}{3}}}$ **23.** $b\left(64^{\frac{1}{12}}a^{\frac{7}{12}}b^{\frac{2}{3}}\right)$ or $2^{\frac{1}{2}}a^{\frac{7}{12}}b^{\frac{5}{3}}$

24. $32^{\frac{1}{6}}x^{\frac{4}{3}}y^{\frac{1}{2}}$ or $2^{\frac{5}{6}}x^{\frac{4}{3}}y^{\frac{1}{2}}$

25. $4y^2\sqrt{2y}$ **26.** $4x\sqrt[4]{y}$

27. $\frac{(\sqrt[4]{a+2})^3}{8}$ **28.** $ab\sqrt[4]{a^2b^2c}$

29. a–b.



c. rotation about the y-axis

30. $\frac{1}{6}$ **31.** 64 **32.** $\frac{9}{4}$
33. $\frac{1}{2}$ **34.** $\frac{1}{2}$ **35.** $\frac{3}{2}$
36. -2 **37.** 2 **38.** 4
39. -1 **40.** $\frac{1}{2}$ **41.** -1
42. \$520.30 **43.** 9.05 mg **44.** 4.7%
45. \$6,553.60 **46.** (0.5, 0.346)

Exploration *(page 316)*

a. $\frac{3^a - 3^{a-2}}{3^{a-1} + 3^a} = \frac{3^a(1 - 3^{-2})}{3^a(3^{-1} + 1)} = \frac{\frac{8}{9}}{\frac{4}{3}} = \frac{8}{12} = \frac{2}{3}$

b. $\frac{4^{a+1} + 4}{2^{a+5} \cdot 2^{a-1} + 16} = \frac{4(4^a + 1)}{2^{2a+4} + 16}$

$= \frac{4(4^a + 1)}{4^{a+2} + 4^2} = \frac{4(4^a + 1)}{4^2(4^a + 1)} = \frac{1}{4}$

Cumulative Review *(pages 316–318)*

Part I

1. 2 **2.** 2 **3.** 3
4. 2 **5.** 1 **6.** 2
7. 2 **8.** 4 **9.** 1
10. 4

Part II

11. The common difference d is 2.
The first term a_1 is 1.

$$a_n = 1 + (n - 1)2 = 2n - 1$$

12.
$$(x - 2)\left(x - \frac{5}{2}\right) = 0$$
$$x^2 - \frac{5}{2}x - 2x + 5 = 0$$
$$x^2 - \frac{9}{2}x + 5 = 0$$
or $2x^2 - 9x + 10 = 0$

Part III

13. $\frac{3 + 2i}{1 - 2i} \cdot \frac{1 + 2i}{1 + 2i}$

$= \frac{3 + 4i - 4}{5}$

$= \frac{-1}{5} + \frac{7}{5}i$

14. $1 + 27^{x+1} = 82$
$27^{x+1} = 81$
$3^{3x+3} = 3^4$
$3^{3x} = 3^1$
$x = \frac{1}{3}$

Part IV

15. *Answer:* $\frac{7 - \sqrt{97}}{2} < x < \frac{7 + \sqrt{97}}{2}$
Use the quadratic formula to find the roots of the corresponding equation:

$$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(1)(-12)}}{2(1)}$$
$$= \frac{7 \pm \sqrt{49 + 48}}{2}$$
$$= \frac{7 \pm \sqrt{97}}{2} \approx -1.42, 8.42$$

Test a number from each interval formed by the roots to find the solution.

16. a. $(x - 2)^2 + y^2 = 16$
b. $x^2 + y^2 - 4x - 12 = 0$
c. $y = x + 2$
d. *Answer:* $(2, 4), (-2, 0)$
Substitute $y = x + 2$ into the equation of the circle:

$$x^2 + (x + 2)^2 - 4x - 12 = 0$$
$$x^2 + x^2 + 4x + 4 - 4x - 12 = 0$$
$$2x^2 - 8 = 0$$
$$2x^2 = 8$$
$$x = \pm 2$$

Substitute $x = \pm 2$ into the equation of the line to find the y-coordinates.

Chapter 8. Logarithmic Functions

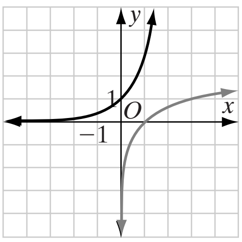
8-1 Inverse of an Exponential Function *(page 323)*

Writing About Mathematics

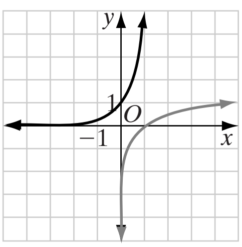
1. Yes. The point $(0, 1)$ is on the graph of any exponential function $y = b^x$. Therefore, since $y = \log_b x$ is the inverse of the exponential function $y = b^x$, $(1, 0)$ is always on its graph.
2. Yes. $x = b^{2y}$, thus $2y = \log_b x$, $y = \frac{1}{2}\log_b x$.

Developing Skills

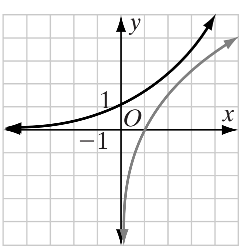
3. a. $f^{-1}(x) = \log_3 x$

b.


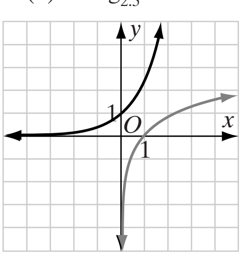
4. a. $f^{-1}(x) = \log_5 x$

b.


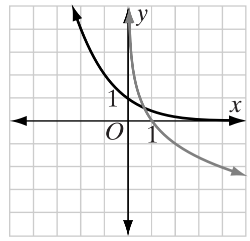
5. a. $f^{-1}(x) = \log_{1.5} x$

b.


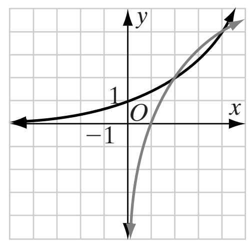
6. a. $f^{-1}(x) = \log_{2.5} x$

b.


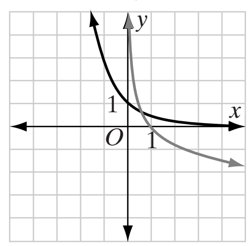
7. a. $f^{-1}(x) = \log_{0.5} x$

b.


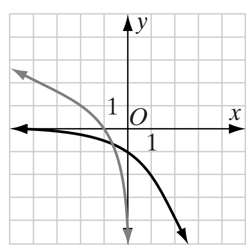
8. a. $f^{-1}(x) = \log_{\sqrt{2}} x$

b.


9. a. $f^{-1}(x) = \log_{\frac{1}{3}} x$

b.


10. a. $f^{-1}(x) = \log_2 (-x)$

b.


11. $y = \log_6 x$
12. $y = \log_{10} x$
13. $y = \log_8 x$
14. $y = \log_{0.1} x$
15. $y = \log_{0.2} x$
16. $y = \log_{\frac{1}{4}} x$
17. $y = \log_{\frac{1}{12}} x$
18. $y = 2^x$
19. $y = 5^x$
20. $y = 10^x$
21. $y = 8^x$
22. $y = (0.1)^x$ or $y = 10^{-x}$

Applying Skills

23. **a.** (0, 1), (1, 3), (2, 9), (3, 27)
b. (1, 0), (3, 1), (9, 2), (27, 3)

24. **a.** (0, 1), (1, 1.05), (2, 1.10), (3, 1.16)
b. (1, 0), (1.05, 1) (1.10, 2), (1.16, 3)

8-2 Logarithmic Form of an Exponential Equation *(pages 326–327)*

Writing About Mathematics

1. $b^{-a} = \frac{1}{b^a}$, so if $b^a = c$, then $b^{-a} = \frac{1}{c}$.

2. If $b^a = c$, then $b^{2a} = c^2$.

Developing Skills

3. $\log_2 16 = 4$	**4.** $\log_5 125 = 3$
5. $\log_8 64 = 2$	**6.** $\log_{12} 1 = 0$
7. $\log_6 216 = 3$	**8.** $\log_{10} 0.1 = -1$
9. $\log_5 0.008 = -3$	**10.** $\log_4 0.0625 = -2$
11. $\log_7 \frac{1}{7} = -1$	**12.** $\log_{64} 4 = \frac{1}{3}$
13. $\log_{625} 125 = \frac{3}{4}$	**14.** $\log_{100} 0.001 = -\frac{3}{2}$
15. $10^2 = 100$	**16.** $5^3 = 125$
17. $4^2 = 16$	**18.** $2^7 = 128$
19. $3^5 = 243$	**20.** $7^0 = 1$
21. $10^{-3} = 0.001$	**22.** $100^{-1} = 0.01$
23. $5^{-2} = 0.04$	**24.** $8^{\frac{1}{3}} = 2$
25. $49^{\frac{3}{2}} = 343$	**26.** $32^{-\frac{2}{5}} = 0.25$

27. 1	**28.** 5	**29.** 3
30. 12	**31.** –1	**32.** −8
33. –2	**34.** −6	**35.** 6
36. 2	**37.** –3	**38.** $-\frac{16}{3}$
39. 4	**40.** 4	**41.** 16
42. 3	**43.** 3	**44.** 5
45. 2	**46.** 4	**47.** 2
48. 2	**49.** −3	**50.** –4
51. 90	**52.** 8	**53.** 8
54. 2	**55.** $\frac{16}{15}$	**56.** $\frac{5}{9}$
57. 1,000	**58.** 5	**59.** 4
60. 2	**61.** 3	**62.** 2
63. $\frac{1}{25}$	**64.** $\frac{1}{390{,}625} = 0.00000256$	
65. $\frac{1}{10}$	**66.** $2\sqrt{2}$	**67.** $\frac{1}{2}$
68. 100	**69.** 4	**70.** $\frac{1}{2}$
71. −3	**72.** $\frac{3}{5}$	**73.** $\frac{1}{10}$
74. 4		

Applying Skills

75. $t = \log_{1.06} A$

76. $n = \log_{0.97} R$

77. **a.** $\log_e A$ or $\ln A$
b. $t = 2{,}500 \log_e A$ or $t = 2{,}500 \ln A$

8-3 Logarithmic Relationships *(pages 331–332)*

Writing About Mathematics

1. $\log_a a^n = n\log_a a = n \cdot 1 = n$ by the logarithm of a power rule and the logarithm of the base rule.

2. No. For example, $\log_{10} 10 \cdot 10 = 2$ and $(\log_{10} 10) \cdot (\log_{10} 10) = 1$.

Developing Skills

3. $\log_3 (27 \times 81) = \log_3 27 + \log_3 81 = 3 + 4 = 7$
$\therefore 27 \times 81 = 3^7 = 2{,}187$

4. $\log_3 (243 \times 27) = \log_3 243 + \log_3 27 = 5 + 3 = 8$
$\therefore 243 \times 27 = 3^8 = 6{,}561$

5. $\log_3 (19{,}683 \div 729) = \log_3 19{,}683 - \log_3 729$
$= 9 - 6 = 3$
$\therefore 19{,}683 \div 729 = 3^3 = 27$

6. $\log_3 (6{,}561 \div 27) = \log_3 6{,}561 - \log_3 27 = 8 - 3 = 5$
$\therefore 6{,}561 \div 27 = 3^5 = 243$

7. $\log_3 9^4 = 4 \log_3 9 = 4 \times 2 = 8$
$\therefore 9^4 = 3^8 = 6{,}561$

8. $\log_3 243^2 = 2 \log_3 243 = 2 \times 5 = 10$
$\therefore 243^2 = 3^{10} = 59{,}049$

9. $2 \log_3 81 + \log_3 9 = 2 \times 4 + 2 = 10$
$\therefore 81^2 \times 9 = 3^{10} = 59{,}049$

10. $\frac{1}{2} \log_3 6{,}561 - \log_3 729 = 4 - 6 = -2$
$\therefore \sqrt{6{,}561} \div 729 = 3^{-2} = \frac{1}{9}$

11. $\frac{1}{4}(\log_3 243 + \log_3 2{,}187) = \frac{1}{4}(5 + 7) = 3$
$\therefore \sqrt[4]{243 \times 2{,}187} = 3^3 = 27$

12. $\frac{1}{2}(\log_3 19{,}683 - \log_3 2{,}187) = \frac{1}{2}(9 - 7) = 1$
$\therefore \sqrt{19{,}683 \div 2{,}187} = 3^1 = 3$

13. $3 \log_3 81 - \frac{1}{2} \log_3 729 = 12 - 3 = 9$
$\therefore 81^3 \div \sqrt{729} = 3^9 = 19{,}683$

14. $\log_3 27 + \frac{1}{3}(\log_3 729 - \log_3 19{,}683)$
$= 3 + \frac{1}{3}(6 - 9) = 2$
$\therefore 27 \times \sqrt[3]{\frac{729}{19{,}683}} = 3^2 = 9$

15. **a.** $\log_3 9$ **b.** 2	**16.** **a.** $\log_3 2{,}187$ **b.** 7
17. **a.** $\log_3 \frac{1}{3}$ **b.** −1	**18.** **a.** $\log_3 27$ **b.** 3
19. **a.** $\log_3 27$ **b.** 3	**20.** **a.** $\log_3 3$ **b.** 1
21. **a.** $4(\log_3 9 - \log_3 27)$ **b.** −4	**22.** **a.** $\frac{1}{2}(\log_3 3 + \log_3 243)$ **b.** 3
23. **a.** $\log_4 16$ **b.** 2	
24. $\log_e 10x$	**25.** $\log_2 ab$
26. $\log_2 (x + 2)^4$	**27.** $\log_{10} \frac{y}{(y - 1)^2}$

28. $\log_e \frac{x \cdot y^2}{z^2}$

29. $\log_3 x^3$

30. $\log_2 2 + \log_2 a + \log_2 b = 1 + \log_2 a + \log_2 b$

31. $\log_3 10 - \log_3 x$

32. $-5 \log_5 a$

33. $2 \log_{10} (x + 1)$

34. $6 \log_4 x - 5 \log_4 y$

35. $\frac{1}{2} \log_e x$

36. $A + B$

37. $2A + B$

38. $3(A + 3)$

39. $A + 3B$

40. $A - B$

41. $2A - 3B$

42. $\frac{1}{2}(A + B)$

43. $A + \frac{1}{2}B$

44. $\frac{1}{2}A - 3B$

45. $\frac{A - B}{2}$

46. $\frac{3}{2}A$

47. $\frac{1}{4}B$

48. 32

49. 32

50. 32

51. 25

52. 2

53. 10

8-4 Common Logarithms *(pages 335–336)*

Writing About Mathematics

1. $\log 80 = \log (10 \times 8) = \log 10 + \log 8 = 1 + \log 8$

2. 10 must be raised to a negative power to yield values less than 1.

Developing Skills

3. 0.57

4. 0.93

5. 1.68

6. 1.75

7. 2.75

8. 3.75

9. −0.47

10. −1.12

11. 0

12. 1

13. 2

14. −1

15. 3.01

16. 1.90

17. −2.70

18. 1.43

19. 0.60

20. 79.59

21. 2.58

22. 1.57

23. −1.49

24. 3.7905

25. 6.7562

26. 24.1324

27. 60.1174

28. 159.5879

29. 364.6700

30. 66,069.3448

31. 0.2902

32. 0.8764

33. 0.0701

34. 0.0010

35. 0.0001

36. $x + y$

37. $2x$

38. $2y$

39. $2x + y$

40. $x + 2y$

41. $3x$

42. $-x$

43. $-y$

44. $-2y$

45. $2x - 1$

46. $x - y$

47. $2(x - y)$

48. $2c$

49. $1 + c$

50. $2 + c$

51. $c - 1$

52. $2 - c$

53. $2c - 1$

54. $2c - 2$

55. $\frac{1}{2}c$

56. $\log \frac{8^2 \cdot (x^2 - 4)}{6} = \log \frac{32(x^2 - 4)}{3}$

57. $\frac{1}{2} \log x + \frac{1}{2} \log y - \frac{1}{2} \log z$

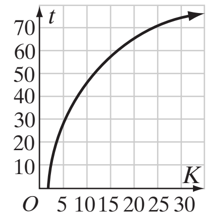
Applying Skills

58. a.

K	1	2	3	4	5
t	0	15.403	24.414	30.807	35.765

K	10	20	30
t	51.169	66.572	75.582

b.



c. Double in the 15th year, triple in the 24th year

59. a. 7.40 **b.** 2.19 **c.** 4.40

8-5 Natural Logarithms *(pages 338–339)*

Writing About Mathematics

1. The bases of the logarithms do not affect the answer. If $\log_a x = y$ and $\log_b x = z$, then $a^y = b^z = x$.

2. $a = 1$. $\log_b 1 = 0$ for any positive $b \neq 1$.

Developing Skills

3. 1.32

4. 2.15

5. 3.87

6. 4.03

7. 6.33

8. 8.63

9. −1.07

10. −2.58

11. 0

12. 1

13. 2

14. −1

15. −0.69

16. 3.56

17. 0.55

18. 13.82

19. 0.39

20. 0.35

21. 1.7837

22. 2.2926

23. 3.9852

24. 5.9239

25. 9.0521

26. 12.9604

27. 123.9651

28. 0.5843

29. 0.9443

30. 0.3152

31. 0.3679

32. 0.1353

33. $x + y$

34. $2y$

35. $2x$

36. $2x + y$

37. $3x + y$

38. $2x + 2y$

39. $-y$

40. $-x$

41. $-x - y$

42. $-2(x + y)$

43. $x - y$

44. $2(x - y)$

45. $2c$

46. $3c$

47. $-c$

48. $-c$

49. $-2c$

50. $-2c$

51. $\frac{1}{2}c$

52. $\frac{1}{2}c$

53. 3.555

54. 5.380

55. 0.693

56. { }

57. $\ln \frac{\sqrt{x} \cdot z^3}{y}$

58. $2 \ln e + \ln x + \frac{1}{2} \ln y - \ln z$
$= 2 + \ln x + \frac{1}{2} \ln y - \ln z$

Hands-On Activity: The Change of Base Formula

a. 2.11 **b.** 4.09 **c.** 1.16
d. 1.84 **e.** −0.73 **f.** −2.58
g. 4.95 **h.** −0.39

8-6 Exponential Equations *(pages 343–344)*

Writing About Mathematics

1. No. You must take the logarithm of each side, not each term. This equation can be solved by first subtracting 6 from each side and then taking the log of both sides.
2. No. The exponent is applied only to 3, not to the entire left side.

Developing Skills

3. 2.26 **4.** 4.17 **5.** 2.86
6. 1.70 **7.** 2.23 **8.** 4.08
9. 6.86 **10.** 2.38 **11.** −0.5
12. −0.15 **13.** 2.14 **14.** 2.89

Applying Skills

15. 15 years old **16.** 5 years
17. 25.5 years **18.** 25 years
19. a. 66 minutes **b.** 22 minutes
20. a. −0.00251 **b.** 828 days

8-7 Logarithmic Equations *(page 346)*

Writing About Mathematics

1. No. The left side must first be combined using the rules for logarithms: $\log x + \log 12 = \log 12x$. Thus, the equation can be solved by writing $12x = 9$.
2. Yes. Taking the logarithm of a number equal to the base is equivalent to 1. Then $\log x = \log (10 \cdot 5)$.

Developing Skills

3. 25 **4.** 6 **5.** 1.5
6. 30 **7.** 126 **8.** 192
9. 4 **10.** 9 **11.** 3
12. 4 **13.** 5 **14.** 0.5
15. 500 **16.** 1 **17.** {1.38, 3.62}
18. 3.65

Review Exercises *(pages 348–350)*

1. a. Graph **b.** $\{x : x > 0\}$
c. {all real numbers} **d.** Graph
e. $y = 3^x$
2. $y = 6^x$ **3.** $y = 2.5^x$
4. $y = \frac{\log x}{\log 82}$ or $y = \log_{82} x$ **5.** $\log_2 8 = 3$
6. $\log_6 36 = 2$ **7.** $\log_{10} 0.1 = -1$
8. $\log_3 \sqrt{3} = \frac{1}{2}$ **9.** $\log_8 4 = \frac{2}{3}$
10. $\log_2 \frac{1}{4} = -2$ **11.** $3^4 = 81$
12. $5^3 = 125$ **13.** $4^{\frac{3}{2}} = 8$
14. $7^{\frac{1}{2}} = \sqrt{7}$ **15.** $10^{-1} = 0.1$
16. $e^0 = 1$ **17.** 9
18. $\frac{56}{9}$ **19.** −12
20. $-\frac{1}{4}$ **21.** 3
22. −2 **23.** $\frac{5}{2}$
24. 4 **25.** $a + b$
26. $2a$ **27.** $a + 2b$
28. $\frac{1}{2}(a + 2b)$ **29.** $\frac{1}{2}b - a$
30. $2(b - a)$ **31.** $\frac{1}{3}(a - b)$
32. $2a - \frac{3}{2}b$ **33.** 1
34. 2.5 **35.** $\frac{5}{2}$
36. 9 **37.** $\frac{1}{36}$
38. −3 **39.** $\sqrt{5}$
40. $\frac{1}{4}$
41. a. $\log_4 \frac{\sqrt[4]{81}}{48}$
b. −2
42. a. $\log_{360} (5 \times 12 \times 6) = \log_{360} 360$
b. 1
43. a. $\log_{0.5} \frac{16}{256} = \log_{0.5} \frac{1}{16}$
b. 4
44. a. $\log_{1.5}\left(\frac{3}{2} \times 3 \times \frac{1}{2}\right) = \log_{1.5} \frac{9}{4}$
b. 2
45. a. $2 \ln 42 - \ln 3$
b. 6.38
46. a. $2 \ln 14 + \ln 0.625$
b. 4.81
47. a. $4 \ln 0.25 - \ln 26 + 5 \ln 3$
b. −3.31
48. $A = xy$ **49.** $A = \frac{x}{y}$
50. $A = \frac{y}{x}$ **51.** $A = x \cdot y^3$
52. $A = \left(\frac{x}{y}\right)^2$ **53.** $A = x \cdot y^{\frac{1}{3}}$ or $A = x\sqrt[3]{y}$
54. 3.5 **55.** 3 **56.** 0.75
57. 3 **58.** 2 **59.** 1.24
60. 2,013 **61.** 16 **62.** 4,500

Exploration *(pages 350–351)*

Steps 1–8.

a_n	g_n
0	1
1	10
1.125	**13.3352143**
1.25	**17.7827941**
1.5	**31.6227766**
2	100

Step 9. $\log(13.3352143) \approx 1.125$
Step 10. The mean of 1.25 and 1.5 is 1.375. Thus, $\log(23.71373706) \approx 1.375$.

Cumulative Review *(pages 351–352)*

Part I

1. 4 **2.** 2 **3.** 4
4. 4 **5.** 4 **6.** 2
7. 4 **8.** 3 **9.** 2
10. 3

Part II

11.
$$f(x) = 4x^3 - x = 0$$
$$x(4x^2 - 1) = 0$$
$$x = 0 \quad \Big| \quad 4x^2 = 1, \quad x = \pm\tfrac{1}{2}$$

12. *Answer:* $x^2 - 10x + 34 = 0, 5 \pm 3i$
Let $a = 1$.
$r_1 + r_2 = -b = 10$
$r_1r_2 = c = 34$

Part III

13.
$$\begin{aligned} 5^{3x} &= 1{,}000 \\ \log 5^{3x} &= \log 1{,}000 \\ 3x \log 5 &= 3 \\ x &= \tfrac{1}{\log 5} \approx 1.43 \end{aligned}$$

14.
$$\begin{aligned} &\sqrt{200} + \sqrt{50} + 2\sqrt{8} \\ &= 10\sqrt{2} + 5\sqrt{2} + 4\sqrt{2} \\ &= 19\sqrt{2} \end{aligned}$$

Part IV

15. a. $30 + 6 + 1.2 + 0.24 + \cdots + 0.000384$
b. 37.499904

16.
$$\begin{aligned} 9.25 &= 15e^{-0.000124t} \\ \ln 9.25 &= \ln 15 - 0.000124t \\ t &= \tfrac{\ln 15 - \ln 9.25}{-0.000124} \\ t &\approx 3{,}900 \text{ years} \end{aligned}$$

Chapter 9. Trigonometric Functions

9-1 Trigonometry of the Right Triangle *(pages 356–357)*

Writing About Mathematics

1. They are equal. By definition, if A is an angle on a right triangle, then $\sin A = \frac{a}{h}$ and $\cos(90 - A) = \frac{a}{h}$.
2. Yes. Since lengths are positive values and the length of a leg of a right triangle is always smaller than the length of the hypotenuse, sin A is a positive value less than 1.

Developing Skills

3. a. $\frac{3}{5}$ **b.** $\frac{4}{5}$ **c.** $\frac{3}{4}$
4. a. $\frac{5}{13}$ **b.** $\frac{12}{13}$ **c.** $\frac{5}{12}$
5. a. $\frac{11}{61}$ **b.** $\frac{60}{61}$ **c.** $\frac{11}{60}$
6. a. $\frac{8}{17}$ **b.** $\frac{15}{17}$ **c.** $\frac{8}{15}$
7. a. $\frac{8}{17}$ **b.** $\frac{15}{17}$ **c.** $\frac{8}{15}$
8. a. $\frac{\sqrt{5}}{5}$ **b.** $\frac{2\sqrt{5}}{5}$ **c.** $\frac{1}{2}$
9. a. $\frac{\sqrt{2}}{3}$ **b.** $\frac{\sqrt{7}}{3}$ **c.** $\frac{\sqrt{14}}{7}$
10. a. $\frac{2}{3}$ **b.** $\frac{\sqrt{5}}{3}$ **c.** $\frac{2\sqrt{5}}{5}$
11. The triangles are similar and therefore have the same trig ratios.
12. $\sin 45° = \cos 45° = \frac{\sqrt{2}}{2}$, $\tan 45° = 1$
13. $\sin 45° = \frac{1}{2}$, $\cos 45° = \frac{\sqrt{3}}{2}$, $\tan 45° = \frac{\sqrt{3}}{3}$

Applying Skills

14. $\sin = \frac{4}{5}$, $\cos = \frac{3}{5}$, $\tan = \frac{4}{3}$
15. 0.25
16. $\sin = \frac{5}{13}$, $\cos = \frac{12}{13}$, $\tan = \frac{5}{12}$
17. 15 m **18.** 125 ft **19.** 56 ft

9-2 Angles and Arcs as Rotations *(pages 360–361)*

Writing About Mathematics

1. Yes, $810° = 90° + (2)360°$.
2. No, two angles that add to 360 are not necessarily coterminal. For example, 150° and 210°.

Developing Skills

In 3–7, answers will be graphs.

3. In quadrant I **4.** Same as 180°
5. Same as 180° **6.** Same as 240°
7. In quadrant II **8.** I
9. II **10.** III **11.** IV
12. IV **13.** II **14.** I
15. IV **16.** I **17.** IV
18. 30° **19.** 52° **20.** 280°
21. 350° **22.** 275° **23.** 90°

24. 220° **25.** 180° **26.** 0°
27. 260°

Applying Skills

28. Clockwise **29.** Counterclockwise
30. a. Clockwise **b.** 2,340°
31. 60 **32.** 12.5
33. a. 87 s **b.** 6 min, 15 sec
34. a. 18° per second **b.** 540° per second **c.** 3,600° per second

9-3 The Unit Circle, Sine, and Cosine *(page 366)*

Writing About Mathematics

1. Since P is a point on the unit circle, the largest value for either x or y is 1 and the smallest value is -1.
2. No. For example, $\sin 45° = \sin 135°$.

Developing Skills

3. a. $\frac{4}{5}$ **b.** $\frac{3}{5}$ **c.** I
4. a. -0.8 **b.** 0.6 **c.** IV
5. a. $\frac{1}{2}$ **b.** $-\frac{\sqrt{3}}{2}$ **c.** II
6. a. $-\frac{2\sqrt{5}}{5}$ **b.** $\frac{\sqrt{5}}{5}$ **c.** IV
7. a. $-\frac{12}{13}$ **b.** $-\frac{5}{13}$ **c.** III
8. a. $-\frac{7}{25}$ **b.** $\frac{24}{25}$ **c.** IV
9. a. $\frac{\sqrt{2}}{2}$ **b.** $\frac{\sqrt{2}}{2}$ **c.** I
10. a. $\frac{40}{41}$ **b.** $-\frac{9}{41}$ **c.** II
11. 90° **12.** 270°
13. 0° **14.** 180°
15. (0, 1) **16.** (−1, 0)
17. (0.2, 1.0) **18.** (−0.7, 0.7)
19. (−0.7, −0.8) **20.** (0.7, −0.7)
21. (−1, 0) **22.** (−0.7, 0.7)
23. a. $\pm\frac{\sqrt{5}}{3}$
b. $\pm\frac{\sqrt{5}}{3}$
c. $\frac{2}{3}$

Applying Skills

24. a. $(5 \cos \theta, 5 \sin \theta)$
b. $(-5 \cos \theta, -5 \sin \theta)$
c. $m\angle ROP' = \theta, m\angle ROP'' = 180 + \theta$
25. a. $(-\cos \theta, \sin \theta)$
b. θ
c. $180 - \theta$

Hands-On Activity

3. Answers will vary: (0.94, 0.34)
5. The values are about the same.
6. Answers will vary: 70° (0.34, 0.94); 100° (−0.17, 0.98); 165° (−0.97, 0.26); 200° (−0.94, −0.34); 250° (−0.34, −0.94); 300° (0.50, −0.87); 345° (0.97, −0.26)

In each case, the values of the sine and cosine are approximately equal to the coordinates of P.

Hands-On Activity: Finding Sine and Cosine Using Any Point on the Plane

(1) $r = 5$, $\sin \theta = \frac{4}{5}$, $\cos \theta = \frac{3}{5}$
(2) $r = 13$, $\sin \theta = \frac{12}{13}$, $\cos \theta = \frac{-5}{13}$
(3) $r = 17$, $\sin \theta = \frac{-15}{17}$, $\cos \theta = \frac{8}{17}$
(4) $r = \sqrt{53}$, $\sin \theta = \frac{-7\sqrt{53}}{53}$, $\cos \theta = \frac{-2\sqrt{53}}{53}$
(5) $r = 5\sqrt{2}$, $\sin \theta = \frac{7\sqrt{2}}{10}$, $\cos \theta = \frac{\sqrt{2}}{10}$
(6) $r = 5\sqrt{2}$, $\sin \theta = \frac{7\sqrt{2}}{10}$, $\cos \theta = \frac{-\sqrt{2}}{10}$
(7) $r = \sqrt{5}$, $\sin \theta = \frac{-2\sqrt{5}}{5}$, $\cos \theta = \frac{\sqrt{5}}{5}$
(8) $r = 3\sqrt{2}$, $\sin \theta = \frac{-\sqrt{2}}{2}$, $\cos \theta = \frac{-\sqrt{2}}{2}$

9-4 The Tangent Function *(pages 372–373)*

Writing About Mathematics

1. a. 45° and 225°. If P is a point on the unit circle and on a 45° angle in standard position, an isosceles right triangle is formed by the x- and y-coordinates of P. Thus, the x- and y-coordinates of P are equal and the sine and cosine of 45° are equal. A similar result holds for 225° by symmetry.
b. 45° and 225°. Since $\frac{\sin \theta}{\cos \theta} = \tan \theta$, if $\tan \theta = 1$, then $\sin \theta = \cos \theta$.
2. $\cos \theta = 0$. Since $\frac{\sin \theta}{\cos \theta} = \tan \theta$, $\tan \theta$ is undefined when the denominator is zero.

Developing Skills

3. a. $\frac{4}{5}$ **b.** $\frac{3}{5}$ **c.** $\frac{4}{5}$ **d.** $\frac{4}{3}$
4. a. $-\frac{12}{13}$ **b.** $\frac{5}{13}$ **c.** $-\frac{12}{13}$ **d.** $-\frac{12}{5}$
5. a. $\frac{8}{10}$ **b.** $-\frac{3}{5}$ **c.** $\frac{4}{5}$ **d.** $-\frac{4}{3}$
6. a. $-\frac{\sqrt{15}}{4}$ **b.** $-\frac{1}{4}$ **c.** $-\frac{\sqrt{15}}{4}$ **d.** $\sqrt{15}$
7. a. $\frac{1}{2}$ **b.** $-\frac{\sqrt{3}}{2}$ **c.** $\frac{1}{2}$ **d.** $-\frac{\sqrt{3}}{3}$

8. **a.** $\frac{2}{3}$ **b.** $\frac{\sqrt{5}}{3}$ **c.** $\frac{2}{3}$ **d.** $\frac{2\sqrt{5}}{5}$

9. **a.** $-\frac{\sqrt{2}}{2}$ **b.** $-\frac{\sqrt{2}}{2}$ **c.** $-\frac{\sqrt{2}}{2}$ **d.** 1

10. **a.** $-\frac{2\sqrt{6}}{5}$ **b.** $\frac{1}{5}$ **c.** $-\frac{2\sqrt{6}}{5}$ **d.** $-2\sqrt{6}$

11. **a.** $\frac{\sqrt{7}}{4}$ **b.** $\frac{3}{4}$ **c.** $\frac{\sqrt{7}}{4}$ **d.** $\frac{\sqrt{7}}{3}$

12. **a.** $\frac{\sqrt{2}}{3}$ **b.** $-\frac{\sqrt{7}}{3}$ **c.** $\frac{\sqrt{2}}{3}$ **d.** $-\frac{\sqrt{14}}{7}$

13. **a.** 25 **b.** $\frac{7}{25}$ **c.** $\frac{24}{25}$ **d.** $\frac{24}{7}$

14. **a.** 17 **b.** $\frac{8}{17}$ **c.** $\frac{15}{17}$ **d.** $\frac{15}{8}$

15. **a.** $\sqrt{2}$ **b.** $-\frac{\sqrt{2}}{2}$ **c.** $-\frac{\sqrt{2}}{2}$ **d.** 1

16. **a.** 5 **b.** $-\frac{3}{5}$ **c.** $-\frac{4}{5}$ **d.** $\frac{4}{3}$

17. **a.** $3\sqrt{5}$ **b.** $-\frac{\sqrt{5}}{5}$ **c.** $\frac{2\sqrt{5}}{5}$ **d.** -2

18. **a.** $2\sqrt{10}$ **b.** $-\frac{\sqrt{10}}{10}$ **c.** $\frac{3\sqrt{10}}{10}$ **d.** -3

19. **a.** $4\sqrt{2}$ **b.** $\frac{\sqrt{2}}{2}$ **c.** $-\frac{\sqrt{2}}{2}$ **d.** -1

20. **a.** $3\sqrt{10}$ **b.** $\frac{3\sqrt{10}}{10}$ **c.** $-\frac{\sqrt{10}}{10}$ **d.** $-\frac{1}{3}$

21. I **22.** IV **23.** III

24. I **25.** II **26.** III

27. **a.** 0
b. Undefined
c. Answers will vary: $270° + 360°n$

28. **a.** 0
b. ± 1
c. Answers will vary: any multiple of 180°

29. **a.** ± 1
b. 0
c. Answers will vary: $90° + 180°n$

Applying Skills

30. **a.** $\{\theta : \theta \text{ is any degree angle}\}$
b. $\{y : -1 \leq y \leq 1\}$
c. No, $\tan\theta$ is undefined at $90° + 180°n$.
d. $\{\theta : \theta \neq 90° + 180°n\}$
e. {all real numbers}

31. Apply the Pythagorean Theorem with $x = \cos\theta$ and $y = \sin\theta$.

32. $m = \frac{y}{x} = \frac{\sin\theta}{\cos\theta} = \tan\theta$

Hands-On Activity

3. Answers will vary: 0.34

5. The values are about the same.

6. Answers will vary: 70° (1, 2.75); 100° (1, −5.67); 165° (1, −0.27); 200° (1, 0.36); 250° (1, 2.75); 300° (1, −1.73); 345° (1, −0.27)
In each case, the value of the tangent is approximately equal to the y-coordinate of P.

9-5 The Reciprocal Trigonometric Functions *(pages 377–378)*

Writing About Mathematics

1. Since $\sec\theta = \frac{1}{\cos\theta}$, for $\sec\theta$ to equal one-half, $\cos\theta$ has to equal 2, which is not possible.

2. $\cot 90° = \frac{1}{\tan 90°} = \frac{\cos 90°}{\sin 90°} = \frac{0}{1} = 0$

Developing Skills

3. **a.** 0.8 **b.** 0.6
c. 0.75 **d.** $\frac{5}{3} = 1.\overline{6}$
e. 1.25 **f.** $\frac{4}{3} = 1.\overline{3}$

4. **a.** -0.28 **b.** 0.96
c. $-\frac{7}{24} = -0.291\overline{6}$ **d.** $\frac{25}{24} = 1.041\overline{6}$
e. $-\frac{25}{7} \approx -3.5714$ **f.** $-\frac{24}{7} \approx -3.4286$

5. **a.** $\frac{\sqrt{35}}{6}$ **b.** $-\frac{1}{6}$
c. $-\sqrt{35}$ **d.** -6
e. $\frac{6\sqrt{35}}{35}$ **f.** $-\frac{\sqrt{35}}{35}$

6. **a.** $\frac{\sqrt{3}}{2}$ **b.** $-\frac{1}{2}$
c. $-\sqrt{3}$ **d.** -2
e. $-\frac{2\sqrt{3}}{3}$ **f.** $-\frac{\sqrt{3}}{3}$

7. **a.** $\frac{1}{3}$ **b.** $\frac{2\sqrt{2}}{3}$
c. $\frac{\sqrt{2}}{4}$ **d.** $\frac{3\sqrt{2}}{4}$
e. 3 **f.** $2\sqrt{2}$

8. **a.** $-\frac{2}{3}$ **b.** $-\frac{\sqrt{5}}{3}$
c. $\frac{2\sqrt{5}}{5}$ **d.** $-\frac{3\sqrt{5}}{5}$
e. $-\frac{3}{2}$ **f.** $\frac{\sqrt{5}}{2}$

9. **a.** $\frac{3\sqrt{2}}{5}$ **b.** $-\frac{\sqrt{7}}{5}$
c. $-\frac{3\sqrt{14}}{7}$ **d.** $-\frac{5\sqrt{7}}{7}$
e. $\frac{5\sqrt{2}}{6}$ **f.** $-\frac{\sqrt{14}}{6}$

10. **a.** $-\frac{3}{7}$ **b.** $-\frac{2\sqrt{10}}{7}$
c. $\frac{3\sqrt{10}}{20}$ **d.** $-\frac{7\sqrt{10}}{20}$
e. $-\frac{7}{3}$ **f.** $\frac{2\sqrt{10}}{3}$

11. **a.** 5 **b.** $\frac{5}{4}$
c. $\frac{5}{3}$ **d.** $\frac{3}{4}$

12. **a.** $\sqrt{17}$ **b.** $\frac{\sqrt{17}}{4}$
c. $\sqrt{17}$ **d.** $\frac{1}{4}$

13. a. $3\sqrt{2}$ b. $-\sqrt{2}$ c. $-\sqrt{2}$ d. 1

14. a. $5\sqrt{2}$ b. $-\sqrt{2}$ c. $-\sqrt{2}$ d. 1

15. a. $6\sqrt{2}$ b. $\sqrt{2}$ c. $-\sqrt{2}$ d. -1

16. a. $4\sqrt{5}$ b. $\frac{\sqrt{5}}{2}$ c. $-\sqrt{5}$ d. $-\frac{1}{2}$

17. a. $9\sqrt{2}$ b. $-\sqrt{2}$ c. $\sqrt{2}$ d. -1

18. a. $3\sqrt{10}$ b. $-\sqrt{10}$ c. $\frac{\sqrt{10}}{3}$ d. -3

19. a. $1, -1$ b. 0 c. $1, -1$
20. a. $1, -1$ b. 0 c. $1, -1$
21. a. 0 b. $1, -1$ c. 0
22. $1, -1$ 23. $90°$

Applying Skills

24. 7.5 feet
25. a. $\frac{13}{5}$ b. $\frac{13}{12}$ c. $\frac{5}{12}$
26. a. 12 mi b. 7.5 mi c. 12 mi
27. a. Divide the given equation by $\cos^2\theta$.
b. No. It is true only where $\tan\theta$ and $\sec\theta$ are defined.
c. OT
28. $\cot\theta = \frac{1}{\tan\theta} = \frac{1}{\frac{\sin\theta}{\cos\theta}} = \frac{\cos\theta}{\sin\theta}$, $\sin\theta \neq 0$

9-6 Function Values of Special Angles (pages 380–381)

Writing About Mathematics

1. Yes. If $P' = (x, y)$, then $\cos\theta = \frac{x}{r}$ and $\sin\theta = \frac{y}{r}$.
2. Yes. $\sin^2\theta + \cos^2\theta = 1$

$$\sin^2\theta + a^2 = 1$$
$$\sin\theta = \pm\sqrt{1 - a^2}$$

Developing Skills

3. $\frac{\sqrt{3}}{2}$ 4. $\frac{1}{2}$ 5. 2
6. $\frac{\sqrt{3}}{3}$ 7. $\sqrt{3}$ 8. $\frac{1}{2}$
9. 2 10. $\frac{\sqrt{3}}{2}$ 11. $\frac{2\sqrt{3}}{3}$
12. $\sqrt{3}$ 13. $\frac{\sqrt{3}}{3}$ 14. $\frac{\sqrt{2}}{2}$
15. $\sqrt{2}$ 16. $\frac{\sqrt{2}}{2}$ 17. $\sqrt{2}$
18. 1 19. 1 20. -1
21. -1 22. 0 23. Undefined
24. 0 25. Undefined 26. 0
27. Undefined 28. -1 29. -1
30. Undefined 31. 0 32. 1
33. 1 34. $\frac{\sqrt{2}+1}{2}$ 35. 3
36. 1 37. 1 38. $\frac{1}{4}$
39. 1 40. $\frac{1}{2}$ 41. 1
42. $\frac{2\sqrt{3}}{3}$ 43. 1 44. $\frac{1}{4}$

Applying Skills

45. 120 ft $\times$ $120\sqrt{3}$ ft
46. Answers will vary:

$$\sin(30°) + \sin(30°) \stackrel{?}{=} \sin(60°)$$
$$\frac{1}{2} + \frac{1}{2} = \frac{\sqrt{3}}{2}$$
$$1 \neq \frac{\sqrt{3}}{2} \text{ ✗}$$

47. Answers will vary. Example:

$$30° < 60°$$
$$\cos(30°) = \frac{\sqrt{3}}{2} > \cos(60°) = \frac{1}{2}$$

9-7 Function Values from the Calculator (pages 384–385)

Writing About Mathematics

1. $\tan 90°$ is undefined.
2. 400° and 40° are co-terminal angles.

Developing Skills

3. 0.4695 4. 0.8192 5. 4.7046
6. -0.1736 7. 0.1736 8. 0.3640
9. 0.3640 10. -0.2588 11. 0.2588
12. -0.9848 13. 0.9848 14. 0.2679
15. 0.2679 16. -0.5736 17. -0.5736
18. -0.1736 19. -0.1736 20. -0.1736
21. 0.9500 22. 0.8450 23. 38.1885
24. 0.9621 25. -0.1352 26. -0.9048
27. -0.4258 28. 16.1190 29. 3.2361
30. 3.8637 31. 0.5095 32. -5.7588
33. 1.2208 34. -3.7321 35. -1.1034
36. 0.2867 37. 1.6616 38. -4.4454
39. 20° 40. 64° 41. 12°
42. 40° 43. 35° 44. 87°
45. 3° 46. 62° 47. 33°

48. 57° **49.** 46° **50.** 85°
51. 15° 30′ **52.** 74° 30′ **53.** 75° 45′
54. 14° 15′ **55.** 40° 48′ **56.** 49° 12′
57. 82° 15′ **58.** 5° 06′

Applying Skills

59. 17° **60.** 20° 29′

61. 51° 09′, 47° 60′, 80° 60′

62. a. II
b. 143°

63. a. IV
b. 286°

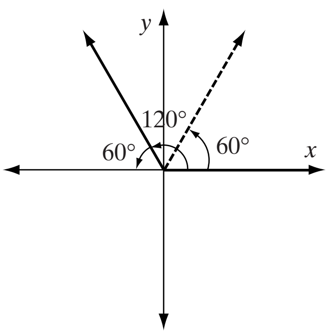
9-8 Reference Angles and the Calculator
(page 391)

Writing About Mathematics

1. Yes, $-\theta$ is equivalent to $360 - \theta$.
2. No. Only sin and tan are negative in quadrant IV. Cos will return a positive value.

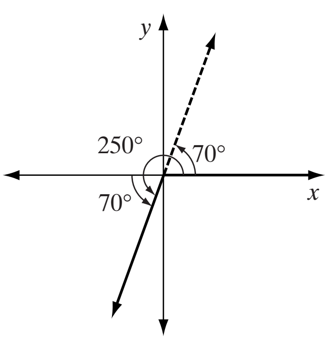
Developing Skills

3. a–c.



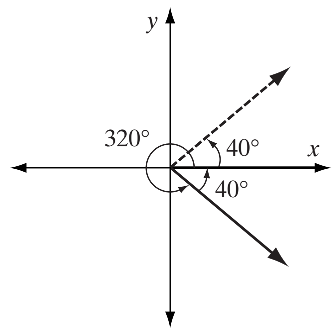
d. 60°

4. a–c.



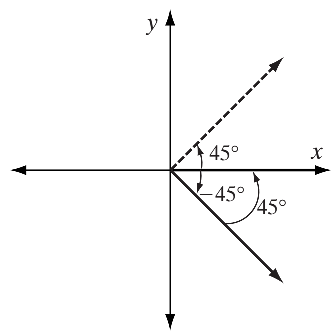
d. 70°

5. a–c.



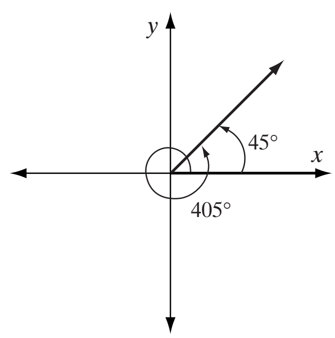
d. 40°

6. a–c.



d. 45°

7. a–c.

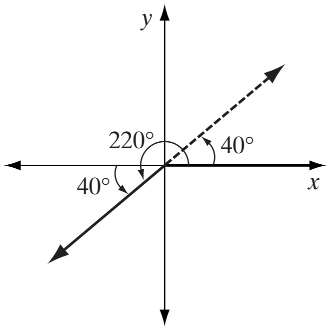


d. 45°

8. 80° **9.** 5° **10.** 30°
11. 70° **12.** 75° **13.** 50°
14. 85° **15.** 70° **16.** 50°
17. 35° **18.** −sin 35° **19.** −cos 85°
20. tan 75° **21.** cos 48° **22.** −tan 10°
23. −sin 75° **24.** −cos 65° **25.** −tan 55°
26. −sin 56° **27.** sin 40° **28.** 20°, 160°
29. 51°, 309° **30.** 18°, 198° **31.** 55°, 235°
32. 54°, 126° **33.** 183°, 357° **34.** 108°, 252°
35. 23°, 337° **36.** 96°, 276° **37.** 14°, 166°
38. 138°, 222° **39.** 188°, 352° **40.** 172°, 352°
41. 0°, 180° **42.** 90°, 270° **43.** 0°, 180°

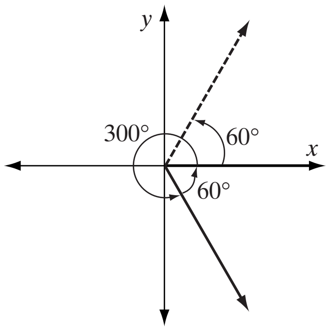
Review Exercises *(pages 394–396)*

1. a–c.



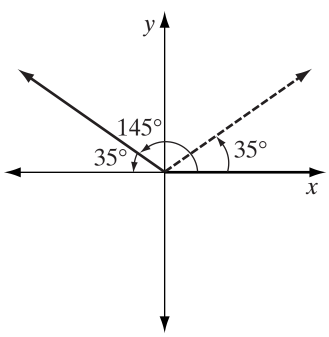
d. 40°

2. a–c.



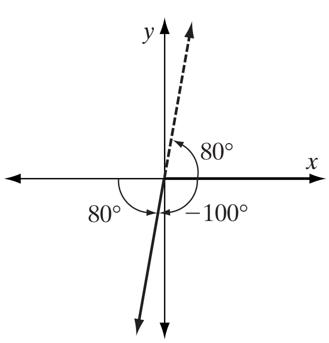
d. 60°

3. a–c.



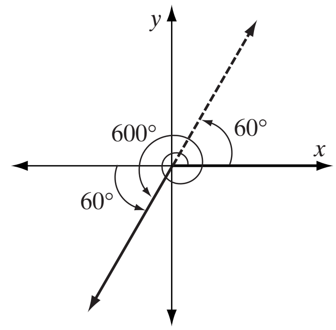
d. 35°

4. a–c.



d. 80°

5. a–c.



d. 60°

6. a. 145°
b. 58°

7. a. IV
b. -0.6
c. 0.8
d. -0.75
e. $-\frac{5}{3} = -1.\overline{6}$
f. 1.25
g. $-\frac{4}{3} = -1.\overline{3}$

8. a. I
b. $\frac{\sqrt{7}}{4}$
c. $\frac{3}{4}$
d. $\frac{\sqrt{7}}{3}$
e. $\frac{4\sqrt{7}}{7}$
f. $\frac{4}{3}$
g. $\frac{3\sqrt{7}}{7}$

9. a. II
b. $\frac{1}{5}$
c. $-\frac{2\sqrt{6}}{5}$
d. $-\frac{\sqrt{6}}{12}$
e. 5
f. $-\frac{5\sqrt{6}}{12}$
g. $-2\sqrt{6}$

10. a. III
b. $-\frac{\sqrt{5}}{3}$
c. $-\frac{2}{3}$
d. $\frac{\sqrt{5}}{2}$
e. $-\frac{3\sqrt{5}}{5}$
f. $-\frac{3}{2}$
g. $\frac{2\sqrt{5}}{5}$

11. a. I
b. $\frac{3}{5}$
c. $\frac{4}{5}$
d. $\frac{3}{4}$
e. $\frac{5}{3}$
f. $\frac{5}{4}$
g. $\frac{4}{3}$

12. a. Quadrantal
b. 0
c. -1
d. 0
e. Undefined
f. -1
g. Undefined

13. **a.** III
b. $-\frac{3}{5}$
c. $-\frac{4}{5}$
d. $\frac{3}{4}$
e. $-\frac{5}{3}$
f. $-\frac{5}{4}$
g. $\frac{4}{3}$

14. **a.** IV
b. $-\frac{13\sqrt{10}}{50}$
c. $\frac{9\sqrt{10}}{50}$
d. $-\frac{13}{9}$
e. $-\frac{5\sqrt{10}}{13}$
f. $\frac{5\sqrt{10}}{9}$
g. $-\frac{9}{13}$

15. **a.** $(0.7, 0.7)$ **b.** $(0.2, 1.0)$

16. $\frac{\sqrt{3}}{2}$ **17.** $\frac{\sqrt{3}}{2}$ **18.** 1
19. $-\sqrt{3}$ **20.** $-\frac{\sqrt{2}}{2}$ **21.** $-\frac{\sqrt{3}}{2}$
22. $\frac{\sqrt{3}}{2}$ **23.** $\frac{\sqrt{3}}{2}$ **24.** $\sqrt{2}$
25. 2 **26.** $\frac{\sqrt{3}}{3}$ **27.** -2
28. 0.6428 **29.** 5.6713 **30.** 0.9397
31. 1.1918 **32.** -0.9925 **33.** -0.4067
34. -0.6428 **35.** 0.1763 **36.** $-\cos 80°$
37. $-\sin 60°$ **38.** $\cos 80°$ **39.** $\tan 30°$
40. $\sin 30°$ **41.** $-\cos 75°$ **42.** $\cos 40°$
43. $\tan 50°$ **44.** 22°, 158° **45.** 24°, 336°
46. 54°, 234° **47.** 224°, 316° **48.** 150°, 330°
49. 145°, 215° **50.** 44°, 316° **51.** 19°, 161°
52. 90°, 270° **53.** 0°, 180° **54.** 90°, 270°
55. 0°, 180°

56. **a.** q **b.** p **c.** t
d. By similar triangles, $\frac{q}{p} = \frac{t}{1}$. In parts **a–c**, we showed that $q = \sin\theta$, $p = \cos\theta$, and $t = \tan\theta$. It follows that $\tan\theta = \frac{\sin\theta}{\cos\theta}$ by substitution.

57. 6° 50′ **58.** 18 feet

Exploration *(page 396)*

In $\triangle OTR$:

$$\sec\theta = \frac{1}{\cos\theta} = \frac{\text{hyp}}{\text{adj}} = \frac{OT}{OR} = \frac{OT}{1} = OT$$

Since $\overline{QS} \parallel \overline{OR}$, $m\angle\theta = m\angle QSO$. Thus:

$$\csc\theta = \csc\angle QSO = \frac{\text{hyp}}{\text{opp}} = \frac{OS}{OQ} = \frac{OS}{1} = OS$$

$$\cot\theta = \cot\angle QSO = \frac{\text{adj}}{\text{opp}} = \frac{QS}{QO} = \frac{QS}{1} = QS$$

Cumulative Review *(pages 396–398)*

Part I

1. 1 **2.** 3 **3.** 2
4. 4 **5.** 2 **6.** 4
7. 1 **8.** 3 **9.** 1
10. 4

Part II

11. $(3 - 2i)(-1 + i) = -3 + 3i + 2i + 2$
$= -1 + 5i$

12.
$$\begin{aligned} |2x - 4| &< 3 \\ -3 < 2x - 4 &< 3 \\ 1 < 2x &< 7 \\ \tfrac{1}{2} < x &< \tfrac{7}{2} \end{aligned}$$

Part III

13. *Answer:* $x = 6$

$$\begin{aligned} 3 + (x + 3)^{\frac{1}{2}} &= x \\ (x + 3)^{\frac{1}{2}} &= x - 3 \\ x + 3 &= x^2 - 6x + 9 \\ 0 &= x^2 - 7x + 6 \\ x &= 1, 6 \end{aligned}$$

Check $x = 1$:
$3 + (1 + 3)^{\frac{1}{2}} \stackrel{?}{=} 1$
$3 + (4)^{\frac{1}{2}} \stackrel{?}{=} 1$
$5 \neq 1$ ✘

Check $x = 6$:
$3 + (6 + 3)^{\frac{1}{2}} \stackrel{?}{=} 6$
$3 + (9)^{\frac{1}{2}} \stackrel{?}{=} 6$
$6 = 6$ ✔

14. Since
$r = \sqrt{(4 - 2)^2 + (1 - 0)^2} = \sqrt{2^2 + 1^2} = \sqrt{5}$,
the equation of the circle is:

$$(x - 2)^2 + (y - 1)^2 = 5$$

Part IV

15. *Answer:* $x = 3 \pm \sqrt{5}$
Write the equation in standard form and then use the quadratic formula with $a = 1$, $b = -6$, and $c = 4$:

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(4)}}{2(1)}$$

$$x = \frac{6 \pm \sqrt{36 - 16}}{2}$$

$$x = \frac{6 \pm \sqrt{20}}{2} = \frac{6 \pm 2\sqrt{5}}{2} = 3 \pm \sqrt{5}$$

16. **a.** $f \circ g(-2) = f(5(-2) + 7) = f(-3)$
$= (-3)^2 + 3 = 12$

b. $f \circ g(x) = f(5x + 7)$
$= (5x + 7)^2 - 5x - 7$
$= 25x^2 + 70x + 49 - 5x - 7$
$= 25x^2 + 65x + 42$

Chapter 10. More Trigonometric Functions

10-1 Radian Measure *(pages 404–406)*

Writing About Mathematics

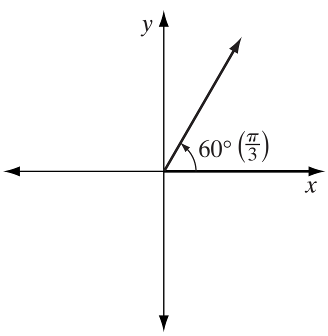
1. Yes, when the length of the intercepted arc is divided by the radius of the circle, the units cancel, giving equivalent ratios.
2. 4π. Two full revolutions is $720° = 4\pi$ radians.

Developing Skills

3. $\frac{\pi}{6}$	**4.** $\frac{\pi}{2}$	**5.** $\frac{\pi}{4}$
6. $\frac{2\pi}{3}$	**7.** $\frac{8\pi}{9}$	**8.** $\frac{3\pi}{4}$
9. $\frac{5\pi}{4}$	**10.** $\frac{4\pi}{3}$	**11.** $\frac{3\pi}{2}$
12. $\frac{11\pi}{6}$	**13.** 60°	**14.** 20°
15. 18°	**16.** 72°	**17.** 200°
18. 270°	**19.** 540°	**20.** 330°
21. 630°	**22.** 57.3°	

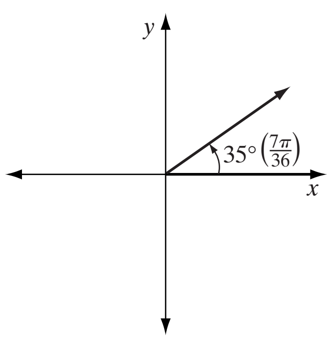
23. a. 60° **b.** $\frac{\pi}{3}$

c.



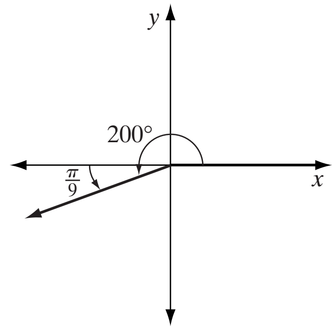
24. a. 35° **b.** $\frac{7\pi}{36}$

c.



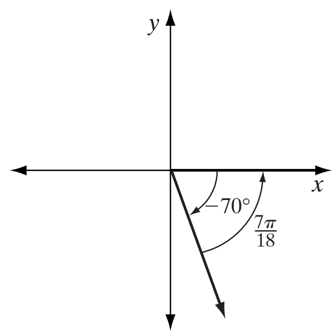
25. a. 200° **b.** $\frac{\pi}{9}$

c.



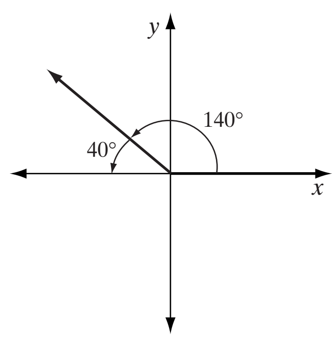
26. a. $-70°$ **b.** $\frac{7\pi}{18}$

c.



27. a. 500° **b.** $\frac{2\pi}{9}$

c.



28. 6	**29.** 2	**30.** 25
31. 3.2	**32.** 6	**33.** 40
34. 5	**35.** 4π	**36.** $\frac{15}{\pi}$
37. 6π	**38.** 3.4 in.	**39.** 2.4
40. 1.5 m		

41.

30°	45°	60°	90°	180°	270°	360°
$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π

Applying Skills

42. 7.2 ft **43.** 9

44. a. Yes, one complete revolution for any circle is 2π radians.

b. No. The radian measure is the same but the length of the radius is not, so the measure of the arc, and therefore the distance traveled, will differ accordingly.

45. 2,457.4 km

10-2 Trigonometric Function Values and Radian Measure *(pages 409–410)*

Writing About Mathematics

1. Yes, $\pi = 180°$ so the formula is correct.

2. Yes. Adding any multiple of $2\pi = 360°$ keeps the terminal side the same.

Developing Skills

3. $\frac{\sqrt{2}}{2}$ **4.** $\sqrt{3}$ **5.** 0

6. $\frac{\sqrt{3}}{3}$ **7.** $-\frac{1}{2}$ **8.** $-\frac{\sqrt{3}}{2}$

9. 1 **10.** 2 **11.** Undefined

12. 1 **13.** 0.2771 **14.** 1.0029

15. 1.3086 **16.** 0.3514 **17.** 0.9732

18. 0.5976 **19.** 0.9912 **20.** 0.5796

21. 1.3785 **22.** 1.2542 **23.** 0.4404

24. 1.3750 **25.** $\frac{1}{2}$ **26.** 0

27. $1 - \frac{\sqrt{2}}{2}$ **28.** $-\frac{5\sqrt{3}}{6}$

Applying Skills

29. $(-0.4236, -0.9059)$

30. a. 2.50 **b.** 2.50

c. $(-0.8011, 0.5985)$ **d.** $(-2.4034, 1.7954)$

e. Since for both points, x is negative and y is positive, P and B are both in quadrant II.

31. a. $166\frac{2}{3}$

b. -0.1615

32. 28.0 ft

33. a. 2,405 ft

b. 2,352 ft

Hands-On Activity 1: The Unit Circle and Radian Measure

1. $(\cos 0, \sin 0) = (1, 0)$;

$\left(\cos\frac{\pi}{6}, \sin\frac{\pi}{6}\right) = \left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$;

$\left(\cos\frac{\pi}{4}, \sin\frac{\pi}{4}\right) = \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$;

$\left(\cos\frac{\pi}{3}, \sin\frac{\pi}{3}\right) = \left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$;

$\left(\cos\frac{\pi}{2}, \sin\frac{\pi}{2}\right) = (0, 1)$

2. $\left(\cos\frac{2\pi}{3}, \sin\frac{2\pi}{3}\right) = \left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$;

$\left(\cos\frac{3\pi}{4}, \sin\frac{3\pi}{4}\right) = \left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$;

$\left(\cos\frac{5\pi}{6}, \sin\frac{5\pi}{6}\right) = \left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$;

$(\cos\pi, \sin\pi) = (-1, 0)$;

$\left(\cos\frac{7\pi}{6}, \sin\frac{7\pi}{6}\right) = \left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$;

$\left(\cos\frac{5\pi}{4}, \sin\frac{5\pi}{4}\right) = \left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$;

$\left(\cos\frac{2\pi}{3}, \sin\frac{2\pi}{3}\right) = \left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$;

$\left(\cos\frac{3\pi}{2}, \sin\frac{3\pi}{2}\right) = (0, -1)$;

$\left(\cos\frac{5\pi}{3}, \sin\frac{5\pi}{3}\right) = \left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$;

$\left(\cos\frac{7\pi}{4}, \sin\frac{7\pi}{4}\right) = \left(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$;

$\left(\cos\frac{11\pi}{6}, \sin\frac{11\pi}{6}\right) = \left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$

Hands-On Activity 2: Evaluating the Sine and Cosine Functions

1. $\sin x$: $\frac{x^9}{9!}, -\frac{x^{11}}{11!}$

$\cos x$: $\frac{x^8}{8!}, -\frac{x^{10}}{10!}$

2. 0.7071

3. 0.5000

10-3 Pythagorean Identities *(pages 413–414)*

Writing About Mathematics

1. Yes. $(\sec\theta)(\csc\theta) = \frac{1}{\cos\theta} \cdot \frac{1}{\sin\theta} = \frac{1}{\cos\theta\sin\theta}$

2. Yes. Both equations are equivalent to the identity $\cos^2\theta + \sin^2\theta = 1$.

Developing Skills

3. $\sin\theta = \frac{1}{5}$, $\cos\theta = -\frac{2\sqrt{6}}{5}$, $\tan\theta = -\frac{\sqrt{6}}{12}$, $\cot\theta = -2\sqrt{6}$, $\sec\theta = -\frac{5\sqrt{6}}{12}$, $\csc\theta = 5$

4. $\sin\theta = \frac{\sqrt{7}}{4}$, $\cos\theta = \frac{3}{4}$, $\tan\theta = \frac{\sqrt{7}}{3}$, $\cot\theta = \frac{3\sqrt{7}}{7}$, $\sec\theta = \frac{4}{3}$, $\csc\theta = \frac{4\sqrt{7}}{7}$

5. $\sin\theta = -\frac{\sqrt{7}}{4}$, $\cos\theta = -\frac{3}{4}$, $\tan\theta = \frac{\sqrt{7}}{3}$, $\cot\theta = \frac{3\sqrt{7}}{7}$, $\sec\theta = -\frac{4}{3}$, $\csc\theta = -\frac{4\sqrt{7}}{7}$

6. $\sin\theta = -\frac{2}{3}$, $\cos\theta = \frac{\sqrt{5}}{3}$, $\tan\theta = -\frac{2\sqrt{5}}{5}$, $\cot\theta = -\frac{\sqrt{5}}{2}$, $\sec\theta = \frac{3\sqrt{5}}{5}$, $\csc\theta = -\frac{3}{2}$

7. $\sin\theta = \frac{2}{3}$, $\cos\theta = -\frac{\sqrt{5}}{3}$, $\tan\theta = -\frac{2\sqrt{5}}{5}$, $\cot\theta = -\frac{\sqrt{5}}{2}$, $\sec\theta = -\frac{3\sqrt{5}}{5}$, $\csc\theta = \frac{3}{2}$

8. $\sin\theta = \frac{2\sqrt{5}}{5}$, $\cos\theta = -\frac{\sqrt{5}}{5}$, $\tan\theta = -2$,
$\cot\theta = -\frac{1}{2}$, $\sec\theta = -\sqrt{5}$, $\csc\theta = \frac{\sqrt{5}}{2}$

9. $\sin\theta = -\frac{4\sqrt{17}}{17}$, $\cos\theta = -\frac{\sqrt{17}}{17}$, $\tan\theta = 4$,
$\cot\theta = \frac{1}{4}$, $\sec\theta = -\sqrt{17}$, $\csc\theta = -\frac{\sqrt{17}}{4}$

10. $\sin\theta = \frac{3\sqrt{7}}{8}$, $\cos\theta = -\frac{1}{8}$, $\tan\theta = -3\sqrt{7}$,
$\cot\theta = -\frac{\sqrt{7}}{21}$, $\sec\theta = -8$, $\csc\theta = \frac{8\sqrt{7}}{21}$

11. $\sin\theta = -\frac{3\sqrt{34}}{34}$, $\cos\theta = -\frac{5\sqrt{34}}{34}$, $\tan\theta = \frac{3}{5}$,
$\cot\theta = \frac{5}{3}$, $\sec\theta = -\frac{\sqrt{34}}{5}$, $\csc\theta = -\frac{\sqrt{34}}{3}$

12. $\sin\theta = \frac{4}{5}$, $\cos\theta = -\frac{3}{5}$, $\tan\theta = -\frac{4}{3}$, $\cot\theta = -\frac{3}{4}$,
$\sec\theta = -\frac{5}{3}$, $\csc\theta = \frac{5}{4}$

13. $\sin\theta = \frac{4}{5}$, $\cos\theta = -\frac{3}{5}$, $\tan\theta = -\frac{4}{3}$, $\cot\theta = \frac{3}{4}$,
$\sec\theta = -\frac{5}{3}$, $\csc\theta = \frac{5}{4}$

14. $\sin\theta = -\frac{\sqrt{37}}{37}$, $\cos\theta = \frac{6\sqrt{37}}{37}$, $\tan\theta = -\frac{1}{6}$,
$\cot\theta = -6$, $\sec\theta = \frac{\sqrt{37}}{6}$, $\csc\theta = -\sqrt{37}$

15. 1 **16.** $\cos\theta$ **17.** $\sin\theta$
18. $\frac{1}{\sin\theta}$ **19.** $\frac{1 + \sin\theta}{\cos\theta}$ **20.** $\frac{\sin\theta}{\cos\theta}$
21. 1 **22.** 0

10-4 Domain and Range of Trigonometric Functions *(page 419)*

Writing About Mathematics

1. Yes, $\cot\theta = \frac{\cos\theta}{\sin\theta} = \cos\theta \cdot \csc\theta$. Both functions are undefined for integer multiples of π.

2. No, $\cot\frac{\pi}{2} = \frac{\cos\frac{\pi}{2}}{\sin\frac{\pi}{2}} = \frac{0}{1} = 0$.

Developing Skills

3. 1 **4.** 0
5. Tangent is undefined at $\frac{\pi}{2} + n\pi$ $(n = 0)$.
6. Secant is undefined at $\frac{\pi}{2} + n\pi$ $(n = 0)$.
7. 1 **8.** 0 **9.** 0
10. Cotangent is undefined at $n\pi$ $(n = 1)$.
11. Secant is undefined at $\frac{\pi}{2} + n\pi$ $(n = 1)$.
12. -1 **13.** 0
14. Cotangent is undefined at $n\pi$ $(n = 0)$.
15. Tangent is undefined at $\frac{\pi}{2} + n\pi$ $(n = -1)$.
16. -1
17. Secant is undefined at $\frac{\pi}{2} + n\pi$ $(n = 5)$.
18. Cotangent is undefined at $n\pi$ $(n = -8)$.
19. Answers will vary: $\frac{\pi}{2} + n\pi$
20. Answers will vary: $n\pi$

10-5 Inverse Trigonometric Functions *(pages 423–425)*

Writing About Mathematics

1. No. The restricted domain of cosine is $0 \le x \le \pi$, while the restricted domain of tangent is $-\frac{\pi}{2} < x < \frac{\pi}{2}$. These two intervals are not equivalent.

2. Yes. The calculator returns an equivalent answer for $\cos^{-1}(-0.5)$ regardless of whether it is in degrees or radians.

Developing Skills

3. a. 30° **b.** $\frac{\pi}{6}$ **4. a.** 45° **b.** $\frac{\pi}{4}$ **5. a.** 0° **b.** 0
6. a. $-45°$ **b.** $-\frac{\pi}{4}$ **7. a.** 120° **b.** $\frac{2\pi}{3}$ **8. a.** $-30°$ **b.** $-\frac{\pi}{6}$
9. a. $-60°$ **b.** $-\frac{\pi}{3}$ **10. a.** $-60°$ **b.** $-\frac{\pi}{3}$ **11. a.** 180° **b.** π
12. a. 90° **b.** $\frac{\pi}{2}$ **13. a.** 0° **b.** 0 **14. a.** 90° **b.** $\frac{\pi}{2}$
15. 37° **16.** 127° **17.** 77°
18. $-77°$ **19.** 26° **20.** 154°
21. 46° **22.** $-46°$ **23.** $-87°$
24. $\frac{\sqrt{2}}{2}$ **25.** 1 **26.** 0
27. -1 **28.** $\frac{-\sqrt{3}}{3}$ **29.** $\frac{1}{2}$
30. -1 **31.** $\frac{\sqrt{2}}{2}$ **32.** $\frac{\sqrt{2}}{2}$
33. $\frac{\pi}{4}$ **34.** $\frac{\pi}{6}$ **35.** $\frac{\pi}{4}$
36. π **37.** $-\frac{\pi}{4}$ **38.** $\frac{\pi}{6}$

39. a. $\theta = \arccos\left(\frac{x}{x + 1}\right)$
b. $\theta = \arcsin\left(\frac{3}{2x + 3}\right)$
c. $\theta = \arctan\left(\frac{x + 1}{x + 2}\right)$

Applying Skills

40. a. $\left[0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \pi\right]$
b. No. The restricted domain of secant does not include $\frac{\pi}{2}$.
c. $(-\infty, -1] \cup [1, \infty)$
d. $(-\infty, -1] \cup [1, \infty)$
e. $\left[0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \pi\right]$

41. a. $\left[-\frac{\pi}{2}, 0\right) \cup \left(0, \frac{\pi}{2}\right]$
b. No. The restricted domain of cosecant does not include 0.
c. $(-\infty, -1] \cup [1, \infty)$
d. $(-\infty, 1] \cup [1, \infty)$
e. $\left[-\frac{\pi}{2}, 0\right) \cup \left(0, \frac{\pi}{2}\right]$

42. a. $\left(-\frac{\pi}{2}, 0\right) \cup \left(0, \frac{\pi}{2}\right)$
b. No. The restricted domain of the tangent function does not include 0.

c. $(-\infty, \infty)$
d. $(-\infty, \infty)$
e. $\left(-\frac{\pi}{2}, 0\right) \cup \left(0, \frac{\pi}{2}\right)$

43. a. $\theta = \arctan \frac{1}{d}$
b. $63°$

10-6 Cofunctions *(pages 427–428)*

Writing About Mathematics

1. Yes. Cofunctions allow you to express any trigonometric function in terms of the sine function. Also, reference angles allow you to express any trigonometric function value in terms of an acute angle.

2. No. If A is in quadrant II, $\cos A = \frac{-\sqrt{3}}{2}$.

Developing Skills

3. a. $\cos 25°$ **b.** 0.9063
4. a. $\sin 10°$ **b.** 0.1736
5. a. $\cot 36°$ **b.** 1.3764
6. a. $\cos 4°$ **b.** 0.9976
7. a. $\sec 42°$ **b.** 1.3456
8. a. $\csc 15°$ **b.** 3.8637
9. a. $\tan 33°$ **b.** 0.6494
10. a. $\sin 20°$ **b.** 0.3420
11. a. $\cos(-20°)$ or $\cos 20°$ **b.** 0.9397
12. a. $\cot(-5°)$ **b.** -11.4301
13. a. $\sin(-40°)$ **b.** -0.6428
14. a. $\csc(-35°)$ **b.** -1.7434
15. a. $\cos(-140°)$ **b.** -0.7660
16. a. $\sin(-165°)$ **b.** -0.2588
17. a. $\cot(-147°)$ **b.** 1.5399
18. a. $\sec(-176°)$ **b.** 1.0024
19. a. $\sin(-210°)$ or $\sin 30°$ **b.** 0.5
20. a. $\cos(-205°)$ **b.** -0.9063
21. a. $\tan(-222°)$ **b.** -0.9004
22. a. $\csc(-195°)$ or $\csc 15°$ **b.** 3.8637
23. $35°$
24. $20°$
25.

$\cos\theta = \sin\left(\frac{\pi}{2} - \theta\right)$	$\sin\theta = \cos\left(\frac{\pi}{2} - \theta\right)$
$\tan\theta = \cot\left(\frac{\pi}{2} - \theta\right)$	$\cot\theta = \tan\left(\frac{\pi}{2} - \theta\right)$
$\sec\theta = \csc\left(\frac{\pi}{2} - \theta\right)$	$\csc\theta = \sec\left(\frac{\pi}{2} - \theta\right)$

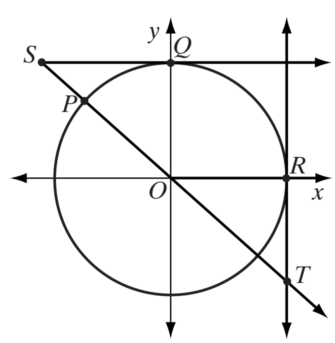
26. a. $\cos \frac{\pi}{6}$ **b.** $\frac{\sqrt{3}}{2}$
27. a. $\sin \frac{\pi}{4}$ **b.** $\frac{\sqrt{2}}{2}$
28. a. $\cot \frac{\pi}{3}$ **b.** $\frac{\sqrt{3}}{3}$
29. a. $\csc\left(-\frac{\pi}{6}\right)$ **b.** -2
30. a. $\sec\left(-\frac{\pi}{3}\right)$ **b.** 2
31. a. $\tan\left(-\frac{\pi}{2}\right)$ **b.** Undefined
32. a. $\cos \frac{3\pi}{4}$ **b.** $-\frac{\sqrt{2}}{2}$
33. a. $\sin\left(-\frac{13\pi}{6}\right)$ **b.** $-\frac{1}{2}$
34. a. $\cot \frac{13\pi}{6}$ **b.** $\sqrt{3}$

Review Exercises *(pages 430–431)*

1. $\frac{5\pi}{12}$
2. $\frac{3\pi}{4}$
3. $\frac{5\pi}{4}$
4. $-\frac{\pi}{3}$
5. $45°$
6. $72°$
7. $210°$
8. $-22.5°$
9. $\frac{\pi}{2}$
10. $\theta = 1$
11. $s = 10$ cm
12. $\theta = 1.5$
13. $r = 4$ cm
14. $\theta = 30$
15. $r = 4$ cm
16. $r = 2.5$ cm
17. $s = 2\pi$ ft
18. a. $90°, 270°$ **b.** $0°, 180°, 360°$ **c.** $90°, 270°$ **d.** $0°, 180°, 360°$
19. a. Domain: $[-1, 1]$, Range: $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
b. Domain: $[-1, 1]$, Range: $[0, \pi]$
c. Domain: {all real numbers}, Range: $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
20. $\frac{1}{2}$
21. $\frac{\sqrt{2}}{2}$
22. $\sqrt{3}$
23. 1
24. $-\frac{1}{2}$
25. -1
26. $-\frac{1}{2}$
27. $-\frac{\sqrt{3}}{2}$
28. -2
29. 2
30. 0
31. -1
32. $2\frac{1}{3}$
33. 12
34. $-\frac{\sqrt{3}}{2}$
35. $-\frac{\sqrt{3}}{3}$
36. Undefined
37. $\sqrt{2}$
38. $-\sqrt{2}$
39. $\frac{1}{2}$

Exploration *(page 431)*

1.



$-OT = \sec\theta$:
Let $T = (0, -t)$.

Let T' = the image of T about a reflection in the x-axis.
Then $OT = OT'$ and $\angle ROT' = 180 - \theta$ is a first-quadrant angle.
From the Chapter 9 Exploration,

$$\sec(180 - \theta) = OT' = OT.$$

Using the properties of reference angles,

$$\sec\theta = -\sec(180 - \theta) = -OT.$$

$OS = \csc\theta$:
Let S' = the image of S about a reflection in the y-axis.
Then $OS = OS'$ and $\angle ROS' = 180 - \theta$ is a first-quadrant angle.
From the Chapter 9 Exploration,

$$\csc(180 - \theta) = OS' = OS$$

Using the properties of reference angles,

$$\csc\theta = \csc(180 - \theta) = OS.$$

$-QS = \cot\theta$:
Let S' = the image of S about a reflection in the y-axis.
Then $QS = QS'$ and $\angle ROS' = 180 - \theta$ is a first-quadrant angle.
From the Chapter 9 Exploration,

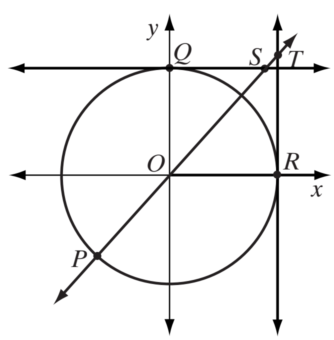
$$\cot(180 - \theta) = QS' = QS$$

Using the properties of reference angles,

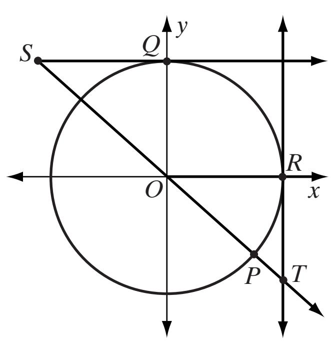
$$\cot\theta = -\cot(180 - \theta) = -QS.$$

A similar procedure can be used to prove steps 2 and 3.

2.



3.



Cumulative Review *(pages 431–433)*

Part I

1. 3 **2.** 4 **3.** 2
4. 3 **5.** 4 **6.** 3
7. 4 **8.** 4 **9.** 2
10. 3

Part II

11. $x = \frac{6 \pm \sqrt{36 - 4(1)(13)}}{2(1)}$

$x = \frac{6 \pm \sqrt{-16}}{2}$

$x = 3 \pm 2i$

12. $280° \cdot \frac{\pi}{180} = \frac{14\pi}{9}$

Part III

13. *Answer:* $(x - 2)^2 + (y - 2)^2 = 80$

The radius of the circle
$= \sqrt{(-2 - 6)^2 + (4 - 0)^2}$
$= \sqrt{8^2 + 4^2}$
$= 4\sqrt{5}$

The center of the circle $= \left(\frac{-2 + 6}{2}, \frac{4 + 0}{2}\right)$
$= (2, 2)$

14. **a.** $\log_{\frac{1}{3}} \frac{\frac{1}{9}}{\left(\frac{1}{27}\right)^2} \cdot \frac{1}{243} = \log_{\frac{1}{3}}\left(\frac{729}{9 \cdot 243}\right) = \log_{\frac{1}{3}} \frac{1}{3}$

b. 1

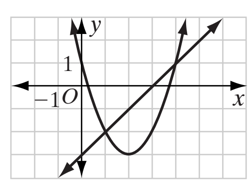
Part IV

15. $\cos\theta = \frac{1}{\sec\theta} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

$\sin\theta = -\sqrt{1 - \cos^2\theta} = -\sqrt{1 - \left(\frac{\sqrt{3}}{3}\right)^2} = -\frac{\sqrt{6}}{3}$

$\tan\theta = -\sqrt{\sec^2\theta - 1} = -\sqrt{(\sqrt{3})^2 - 1}$
$= -\sqrt{2}$

16.



(1, −2), (4, 1)

Chapter 11. Graphs of Trigonometric Functions

11-1 Graph of the Sine Function
(pages 440–441)

Writing About Mathematics

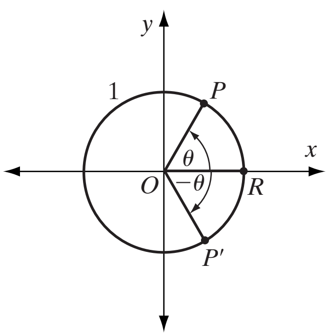
1. Yes, since for each (x, y) on the graph there is also a point $(-x, -y)$ on the graph.
2. Yes. The period of $y = \sin x$ is 2π.

Developing Skills

3. Graph
   a. $0 \leq x < \frac{\pi}{2}, \frac{3\pi}{2} < x < \frac{5\pi}{2}, \frac{7\pi}{2} < x \leq 4\pi$
   b. $\frac{\pi}{2} < x < \frac{3\pi}{2}, \frac{5\pi}{2} < x < \frac{7\pi}{2}$
   c. 2 cycles
4. 1 **5.** -1 **6.** 2π
7. No. It fails the horizontal line test.
8. **a.** $\cos \frac{\pi}{3} = \frac{1}{2}$
   b. $\frac{\pi}{3} + 2n\pi$ or $\frac{2\pi}{3} + 2n\pi$

Applying Skills

9. **a–b.**



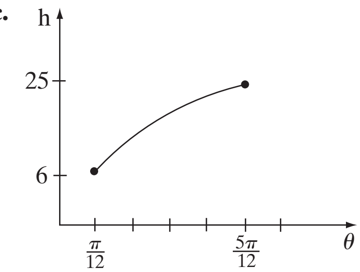
$-\theta$ is the reflection of θ about the x-axis.
$\sin \theta = -\sin(-\theta)$

c. Yes. For all angles in the four quadrants, $\sin \theta = -\sin(-\theta)$.

d. Yes. $\sin 0° = -\sin(-0°) = \sin 180° = -\sin(-180°) = 0$
$\sin 90° = -\sin(-90°) = \sin 270° = -\sin(-270°) = 1$

e. Yes, since for all x in the domain, $f(x) = -f(-x)$.

10. **a.** $\frac{\pi}{12} \leq \theta \leq \frac{5\pi}{12}$
    b. $h(\theta) = 25 \sin \theta$
    c.



d. $25 \sin\frac{5\pi}{12} \approx 24.15$ ft

11. **a.** $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$
    b. $-\pi \leq x \leq \pi$
    c.

x	$\sin x$	$Y_2(x)$	$Y_3(x)$
$\frac{\pi}{6}$	0.5	0.50000213	0.49999999
$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2} \approx 0.70710678$	0.70714305	0.70710647
π	0	0.52404391	-0.0752206

Y_3 always gives the better approximation.

11-2 Graph of the Cosine Function
(pages 445–447)

Writing About Mathematics

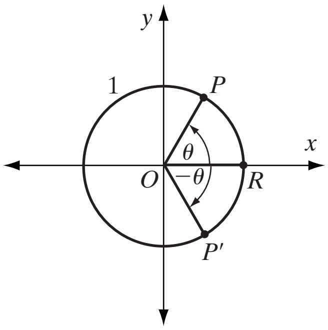
1. Yes. For every (x, y) on the graph there is also a point $(-x, y)$ on the graph.
2. Yes. The period of $y = \sin x$ is 2π.

Developing Skills

3. Graph
   a. $\pi < x < 2\pi, 3\pi < x < 4\pi$
   b. $0 < x < \pi, 2\pi < x < 3\pi$
   c. 2 cycles
4. 1
5. -1
6. 2π
7. No. It fails the horizontal line test.

Applying Skills

8. a–b.



$-\theta$ is the reflection of θ about the x-axis.
$\cos\theta = \cos(-\theta)$

c. Yes. For all angles in the four quadrants, $\cos\theta = \cos(-\theta)$.

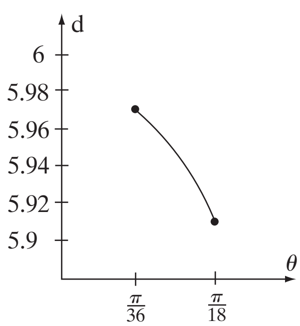
d. Yes. $\cos 0° = \cos(-0°) = \cos 180° = \cos(-180°) = 1$
$\cos 90° = \cos(-90°) = \cos 270° = \cos(-270°) = 0$

e. Yes. For all x in the domain, $f(x) = f(-x)$.

9. a. $d(\theta) = 6\cos\theta$

b. $\frac{\pi}{36} \le \theta \le \frac{\pi}{18}$

c.



d. $6\cos\frac{\pi}{18} \approx 5.901$ feet

10. a. $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$

b. $-\frac{3\pi}{4} \le x \le \frac{3\pi}{4}$

c.

x	$\cos x$	$Y_2(x)$	$Y_3(x)$
$-\frac{\pi}{6}$	$\frac{\sqrt{3}}{2} \approx 0.86602540$	0.86605388	0.86602526
$-\frac{\pi}{4}$	$\frac{\sqrt{2}}{2} \approx 0.70710678$	0.70742921	0.70710321
$-\pi$	-1	0.12390993	-1.211353

Y_3 always gives the better approximation.

11-3 Amplitude, Period, and Phase Shift
(pages 453–455)

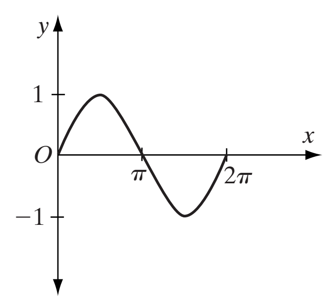
Writing About Mathematics

1. Yes. The first graph is shifted $\frac{\pi}{2}$ to the left and the second is shifted $\frac{\pi}{2}$ to the right, resulting in the graphs starting π units apart. Since this is equal to the period of each curve, their graphs will completely overlap.
2. No. If we factor out a 2 from the second equation, we see that its graph is shifted $\frac{\pi}{8}$ units, in contrast to a shift of $\frac{\pi}{4}$ units for the first graph.

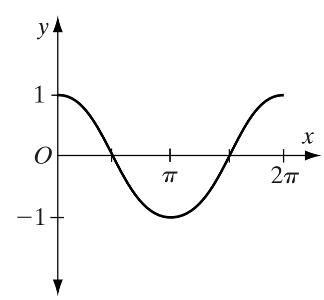
Developing Skills

3. 1 **4.** 2 **5.** 5
6. 3 **7.** $\frac{3}{4}$ **8.** $\frac{1}{2}$
9. 0.6 **10.** $\frac{1}{8}$ **11.** 2π
12. 2π **13.** $\frac{2\pi}{3}$ **14.** π
15. 4π **16.** 6π **17.** $\frac{4\pi}{3}$
18. $\frac{8\pi}{3}$ **19.** $-\frac{\pi}{2}$ **20.** $\frac{\pi}{2}$
21. $-\frac{\pi}{3}$ **22.** $\frac{\pi}{4}$ **23.** $\frac{\pi}{6}$
24. $-\frac{3\pi}{4}$ **25.** $-\pi$ **26.** $\frac{\pi}{2}$

27.



28.



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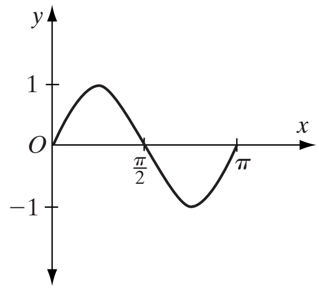

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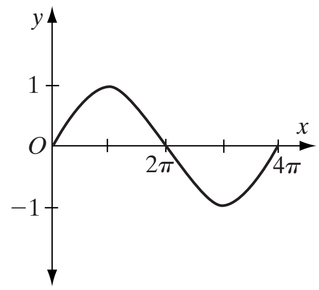
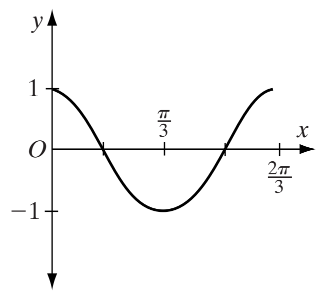
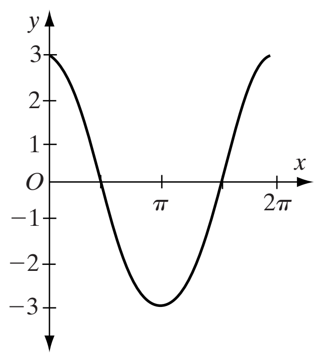


31.



32.



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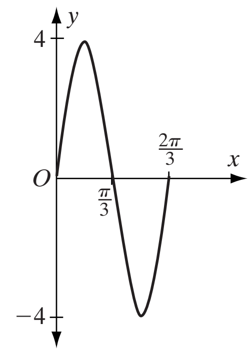

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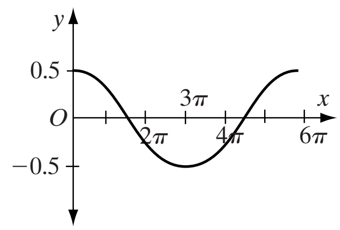

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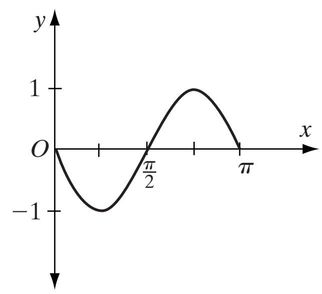

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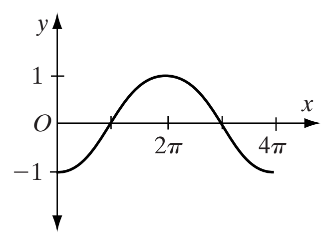

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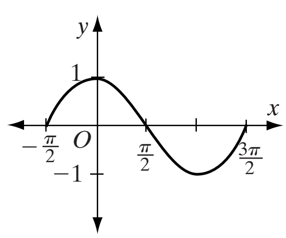
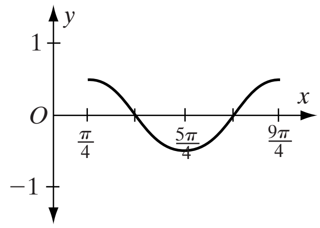


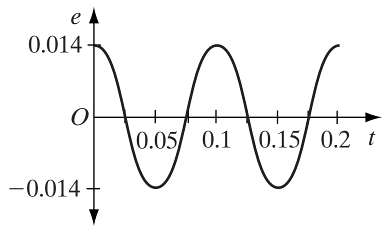
38.



Applying Skills

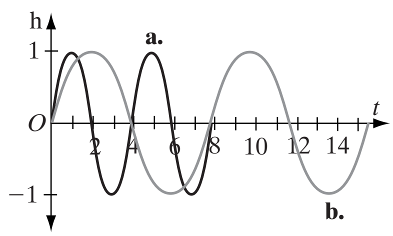
39. Since sine and cosine are cofunctions, it follows that $\sin x = \cos\left(x - \frac{\pi}{2}\right)$.

40. a.



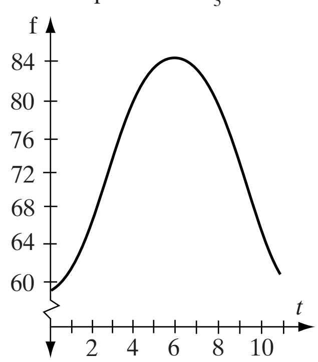
b. 0.014 volt

41. a–b.



c. The period of "middle C" appears to be one-half the period of C_3.

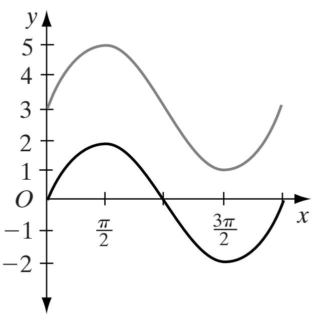
42. a.



b. No. The period of this function is not 12 months, but $\frac{2\pi}{0.5} \approx 12.566$ months. Extending this model would shift all the temperatures by more than half a month each year.

Hands-On Activity

1–2.



3. $y = 3 + 2 \sin x$
4. Maximum = 5, minimum = 1
5. The amplitude is equal to one-half the difference between the maximum and the minimum values: $A = \frac{5-1}{2} = 2$
6. (2) Graph
   (3) $y = -4 + 2 \sin x$
   (4) Maximum = –2, minimum = –6
   (5) The amplitude is equal to one-half the difference between the maximum and minimum values: $A = \frac{(-2)-(-6)}{2} = 2$

11-4 Writing the Equation of a Sine or Cosine Graph *(pages 457–459)*

Writing About Mathematics

1. No. The equation that Tyler wrote has period π and phase shift π. Thus, it is equivalent to $y = 5 \cos 2x$. The maximum of this curve is at $n\pi$ and the minimum is at $\frac{\pi}{2} + n\pi$.
2. Yes. The phase shift of the first graph is equal to the period of both equations.

Developing Skills

3. **a.** $y = \sin x$
   b. $y = \cos\left(x - \frac{\pi}{2}\right)$
4. **a.** $y = \sin\left(x + \frac{\pi}{2}\right)$
   b. $y = \cos x$
5. **a.** $y = 2 \sin\left(x + \frac{\pi}{2}\right)$
   b. $y = 2 \cos x$
6. **a.** $y = 3 \sin 2x$
   b. $y = 3 \cos\left(2x - \frac{\pi}{2}\right)$
7. **a.** $y = \sin \frac{x}{2}$
   b. $y = \cos \frac{1}{2}(x - \pi)$
8. **a.** $y = 2 \sin\left(3x + \frac{\pi}{2}\right)$
   b. $y = 2 \cos 3x$

9. a. $y = \sin\left(x + \frac{\pi}{3}\right)$

b. $y = \cos\left(x - \frac{\pi}{6}\right)$

10. a. $y = 2\sin\left(x + \frac{\pi}{3}\right)$

b. $y = 2\cos\left(x - \frac{\pi}{6}\right)$

11. a. $y = \sin 2\left(x - \frac{\pi}{4}\right)$

b. $y = -\cos 2x$

12. a. $y = 3\sin\frac{1}{2}\left(x + \frac{3\pi}{4}\right)$

b. $y = 3\cos\frac{1}{2}\left(x - \frac{\pi}{4}\right)$

13. a. $y = -\sin 2x$

b. $y = \cos 2\left(x + \frac{\pi}{4}\right)$

14. a. $y = 2\sin\left(x - \frac{\pi}{2}\right)$

b. $y = -\cos x$

Applying Skills

15. a. 0.75 m **b.** 10 s

c. $\frac{1}{10} = 0.1$ cycle per second

d. $h(t) = 0.75\cos\left(\frac{\pi}{5}x\right)$

e. No; if the amplitude is 0.75, then the maximum height is 0.75 meter.

11-5 The Graph of the Tangent Function *(pages 462–463)*

Writing About Mathematics

1. The tangent graph has no maximum or minimum values, the period is π rather than 2π, it has vertical asymptotes, and the range is all real numbers rather than $[-1, 1]$.

2. No; the range is $(-\infty, \infty)$.

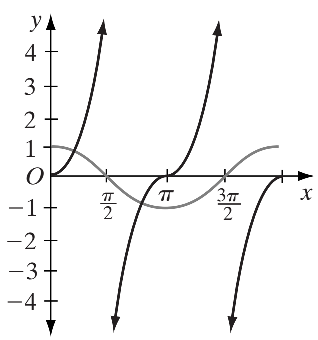
Developing Skills

3. Graph

a. π **b.** $\left\{x: x \neq \frac{\pi}{2} + n\pi\right\}$

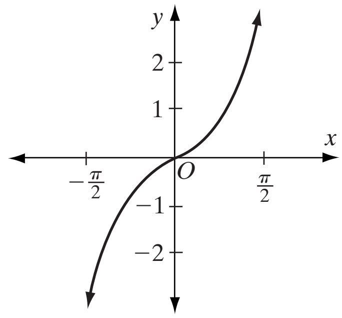
c. $(-\infty, \infty)$

4. a–b.

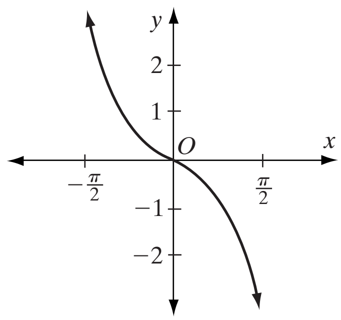


c. 2

5. a.



b–c.



d. They are the same.

Applying Skills

6. a. $h = r\tan\theta$ **b.** $V = \frac{1}{3}\pi r^3 \tan\theta$

7. a. (1) $\frac{\pi}{2}$

(2) $\frac{\pi}{4}$

(3) $AP = r\tan\frac{\pi}{4}$

(4) $AB = s = 2r\tan\frac{\pi}{4}$

(5) Perimeter $= 2(4)r\tan\frac{\pi}{4}$

b. (1) $\frac{2\pi}{5}$

(2) $\frac{\pi}{5}$

(3) $AP = r\tan\frac{\pi}{5}$

(4) $AB = s = 2r\tan\frac{\pi}{5}$

(5) Perimeter $= 2(5)r\tan\frac{\pi}{5}$

c. For any regular polygon with n sides circumscribing a circle of radius r, the perimeter is $2nr\tan\frac{\pi}{n}$.

11-6 Graphs of the Reciprocal Functions *(pages 466–467)*

Writing About Mathematics

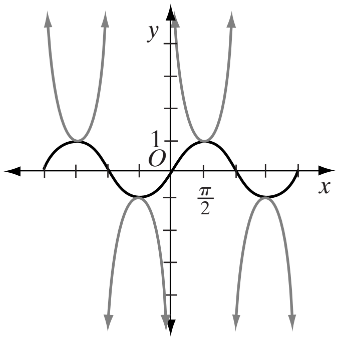
1. Cotangent is the reciprocal of tangent. As the value of tan x increases, its reciprocal decreases, so cot x decreases for all values of x for which it is defined.

2. sec x increases from 1 to ∞ in the interval $\left[0, \frac{\pi}{2}\right)$ and increases from $-\infty$ to -1 in the interval $\left(\frac{\pi}{2}, \pi\right]$.

Developing Skills

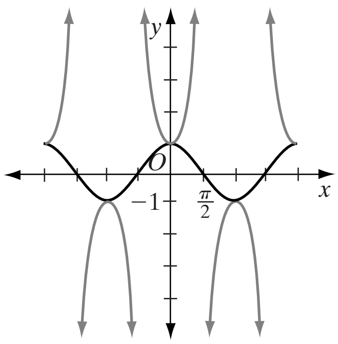
3. (3) **4.** (8) **5.** (6)
6. (7) **7.** (1) **8.** (5)
9. (2) **10.** (4)

11. a.



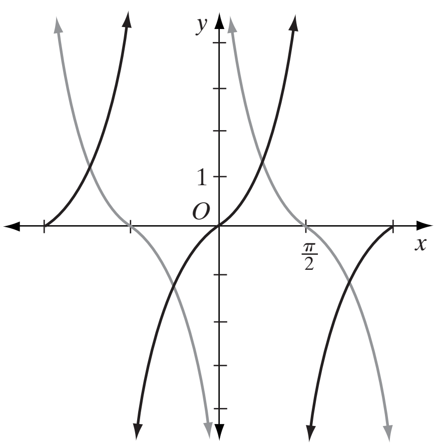
b. $-\frac{3\pi}{2}, -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}$

12. a.



b. $-2\pi, -\pi, 0, \pi, 2\pi$

13. a.



b. $-\frac{3\pi}{4}, -\frac{\pi}{4}, \frac{\pi}{4}, \frac{3\pi}{4}$

14. $-\frac{3\pi}{2}, -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}$
15. $-2\pi, -\pi, 0, \pi, 2\pi$
16. $-\frac{3\pi}{2}, -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}$
17. $-2\pi, -\pi, 0, \pi, 2\pi$
18. $y = \cot x$ and $y = \csc x$
19. $y = \tan x$ and $y = \sec x$
20. a. Odd **b.** Odd **c.** Even **d.** Odd

Applying Skills

21. a. $a = 10 \sec \theta$

b.

θ	$\frac{\pi}{18}$	$\frac{\pi}{9}$	$\frac{\pi}{6}$	$\frac{2\pi}{9}$
a	10.2	10.6	11.5	13.1

c. No **d.** 115.2 ft

11-7 Graphs of Inverse Trigonometric Functions *(pages 471–472)*

Writing About Mathematics

1. Since $\sin(-30°) = -\frac{1}{2}$, $\arcsin\left(-\frac{1}{2}\right) = -30°$. The reference angle of $-30°$ is $30°$.
2. No, $\tan(220°) = 0.839 \neq 1$.

Developing Skills

3. 30° **4.** 60° **5.** 45°
6. 60° **7.** −90° **8.** 90°
9. −60° **10.** 135° **11.** −45°
12. −45° **13.** 0° **14.** 180°
15. $\frac{\pi}{2}$ **16.** 0 **17.** $\frac{\pi}{4}$
18. $\frac{\pi}{3}$ **19.** $-\frac{\pi}{3}$ **20.** $\frac{\pi}{3}$
21. $\frac{2\pi}{3}$ **22.** $\frac{\pi}{3}$ **23.** $-\frac{\pi}{3}$
24. 0 **25.** $\frac{\pi}{2}$ **26.** 0
27. 0 **28.** 0 **29.** 1
30. 0.5 **31.** $-\frac{\sqrt{2}}{2}$ **32.** −0.5

In 33–35, part **a**, answers will be graphs.

33. b. $y = \arcsin x$
Domain $= \{x: -1 \leq x \leq 1\}$
Range $= \left\{y: -\frac{\pi}{2} \leq y \leq \frac{\pi}{2}\right\}$
$y = \sin x$
Domain $= \left\{x: -\frac{\pi}{2} \leq x \leq \frac{\pi}{2}\right\}$
Range $= \{y: -1 \leq y \leq 1\}$

34. b. $y = \arccos x$
Domain $= \{x: -1 \leq x \leq 1\}$
Range $= \{y: 0 \leq y \leq \pi\}$
$y = \cos x$
Domain $= \{x: 0 \leq x \leq \pi\}$
Range $= \{y: -1 \leq y \leq 1\}$

35. b. $y = \arctan x$
Domain $= \{x: x \text{ is a real number}\}$
Range $= \left\{y: -\frac{\pi}{2} < y < \frac{\pi}{2}\right\}$
$y = \tan x$
Domain $= \left\{x: -\frac{\pi}{2} < x < \frac{\pi}{2}\right\}$
Range $= \{y: y \text{ is a real number}\}$

Applying Skills

36. a. $\theta = \tan^{-1}\left(\frac{d}{100}\right)$
b. $\theta = 26.6°$ or $\theta = 0.46$ radians or $\theta = 26° 34'$
c. $71° 34'$

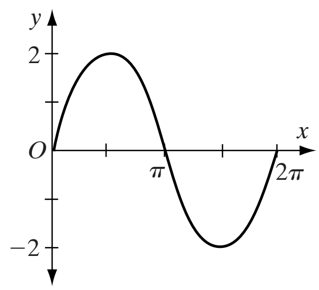
11-8 Sketching Trigonometric Graphs
(pages 474–475)

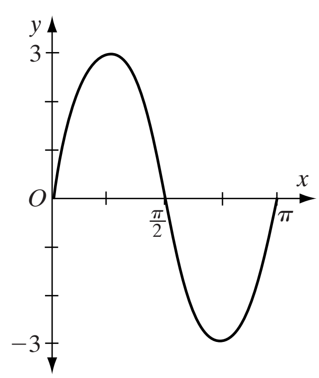
Writing About Mathematics

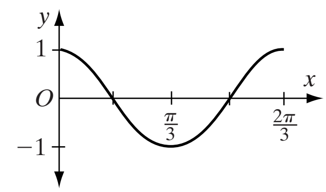
1. Yes. $y = \tan\left(x - \frac{\pi}{4}\right)$ is the tangent graph with a phase shift of $\frac{\pi}{4}$. Therefore, the asymptotes also have a phase shift of $\frac{\pi}{4}$.

2. No. $y = \sin\left(2x - \frac{\pi}{4}\right) = \sin 2\left(x - \frac{\pi}{8}\right)$. The phase shift is $\frac{\pi}{8}$, not $\frac{\pi}{4}$.

Developing Skills

3.


4.


5.


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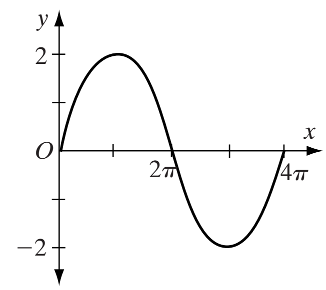

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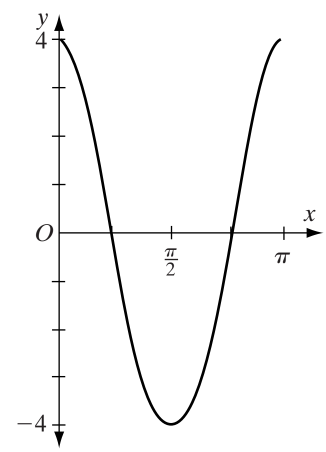

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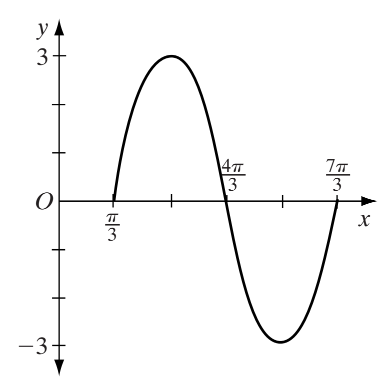

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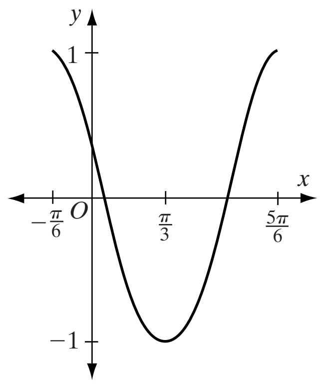

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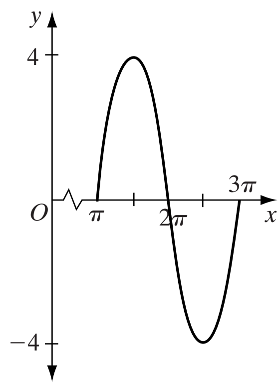
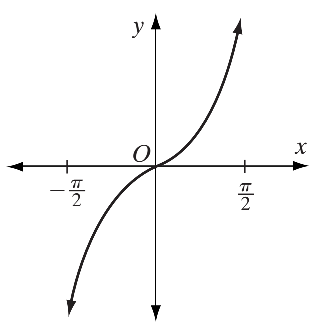
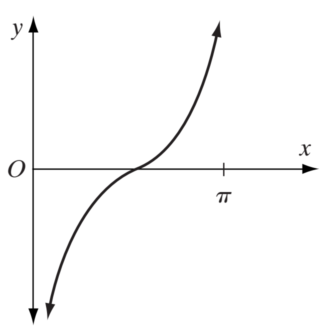


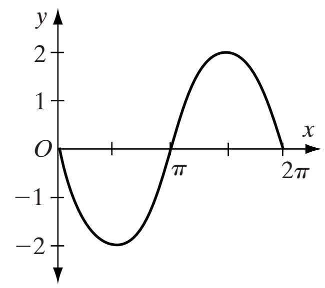
11.



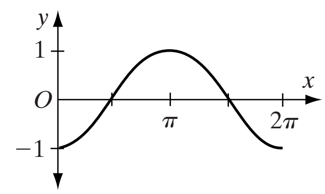
12.



13.



14.



15. a. $y = \sin\left(x + \frac{\pi}{3}\right)$

b. $y = \cos\left(x - \frac{\pi}{6}\right)$

16. a. $y = -2\sin\left(x + \frac{\pi}{3}\right)$

b. $y = -2\cos\left(x - \frac{\pi}{6}\right)$

17. a. $y = 4\sin x$

b. $y = 4\cos\left(x - \frac{\pi}{2}\right)$

18. a. $y = 4\sin\left(x + \frac{\pi}{2}\right)$

b. $y = 4\cos x$

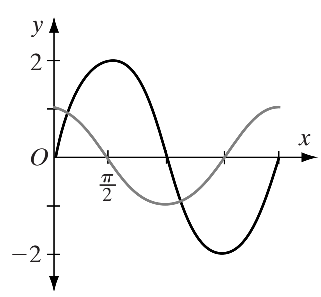
19. a. $y = -4\sin\left(x - \frac{\pi}{3}\right)$

b. $y = 4\cos\left(x + \frac{\pi}{6}\right)$

20. a. $y = -2\sin\left(x - \frac{\pi}{3}\right)$

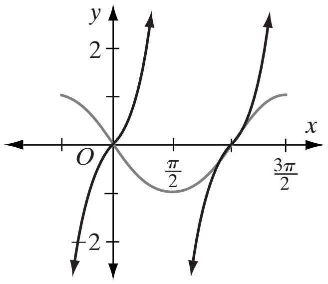
b. $y = 2\cos\left(x + \frac{\pi}{6}\right)$

21. a.



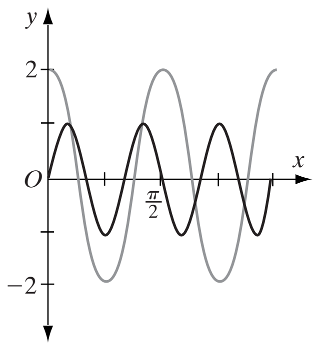
b. 2

22. a.



b. 2

23. a.

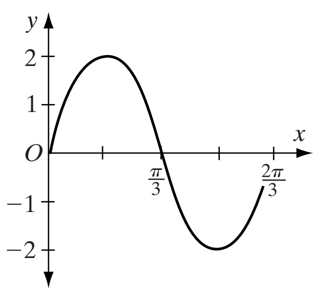


b. 4

Review Exercises *(pages 476–479)*

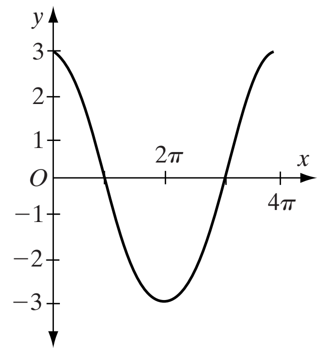
1. a. 2 **b.** $\frac{2\pi}{3}$ **c.** $\frac{3\pi}{2}$
d. {all real numbers} **e.** $[-2, 2]$

f.



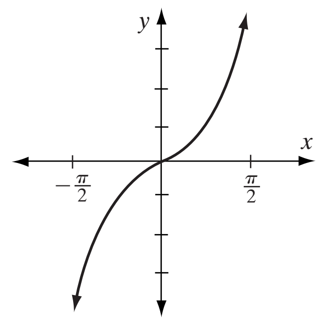
2. a. 3 **b.** 4π **c.** $\frac{1}{4\pi}$
d. {all real numbers} **e.** $[-3, 3]$

f.



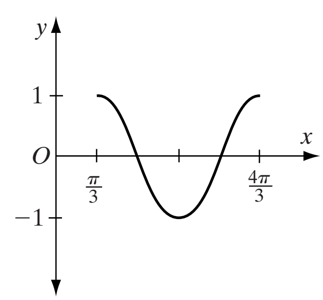
3. a. No amplitude **b.** π
c. $\frac{1}{\pi}$ **d.** $\left\{x: x \neq \frac{\pi}{2} + n\pi\right\}$
e. {all real numbers}

f.



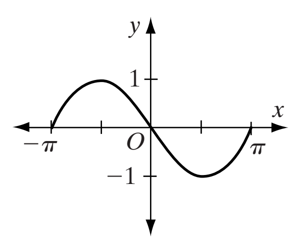
4. a. 1 **b.** π **c.** $\frac{1}{\pi}$
d. {all real numbers} **e.** $[-1, 1]$

f.



5. a. 1 **b.** 2π **c.** $\frac{1}{2\pi}$
d. {all real numbers} **e.** $[-1, 1]$

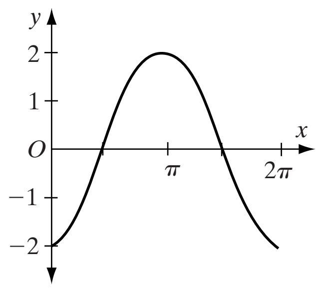
f.



6. **a.** 2 **b.** 2π **c.** $\frac{1}{2\pi}$

d. {all real numbers} **e.** $[-2, 2]$

f.



7. **a.** $y = \sin 2x$

b. $y = \cos 2\left(x - \frac{\pi}{4}\right)$

8. **a.** $y = -2 \sin x$

b. $y = 2 \cos \left(x + \frac{\pi}{2}\right)$

9. **a.** $y = 3 \sin \frac{1}{2}\left(x + \frac{5\pi}{6}\right)$

b. $y = 3 \cos \frac{1}{2}\left(x - \frac{\pi}{6}\right)$

10. **a.** $y = -\sin x$

b. $y = \cos \left(x + \frac{\pi}{2}\right)$

11. (4) **12.** (1) **13.** (2)

14. (3) **15.** $\frac{\pi}{6}$ **16.** $\frac{\pi}{2}$

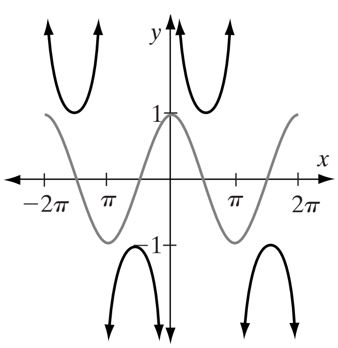
17. $\frac{\pi}{6}$ **18.** $\frac{5\pi}{6}$ **19.** $\frac{\pi}{4}$

20. $-\frac{\pi}{6}$

21. **a.** $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ **b.** $[-1, 1]$

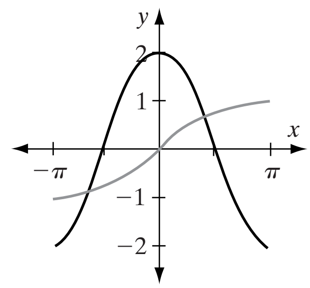
c. $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ **d.** Graph

22. **a–b.** **c.** 0



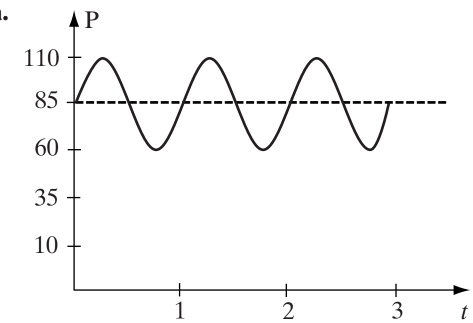
23. $x = -\frac{3\pi}{2}, x = -\frac{\pi}{2}, x = \frac{\pi}{2}, x = \frac{3\pi}{2}$

24. **a.** **b.** 2



25. No. The function $y = \csc x$ is undefined at $x = n\pi$ for integer values of n.

26. **a.**



b. 1 **c.** 25

d. 110 mmHg **e.** 60 mmHg

27. **a.** 4.5 ft **b.** 14 hr

c. $y = 15.5 + 4.5 \sin \frac{\pi}{7}x$

Exploration *(pages 478–479)*

Answers will vary.

Cumulative Review *(pages 479–481)*

Part I

1. 3 **2.** 4 **3.** 3

4. 4 **5.** 2 **6.** 1

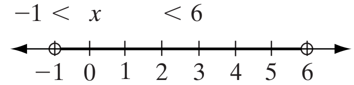
7. 2 **8.** 1 **9.** 1

10. 3

Part II

11.

$$\begin{aligned} |2x - 5| &< 7 \\ -7 < 2x - 5 &< 7 \\ -2 < 2x &< 12 \\ -1 < x &< 6 \end{aligned}$$



12. *Answer:* 123°, 303°

$$\tan\theta = -1.54$$
$$\theta = \tan^{-1}(-1.54)$$
$$\theta \approx -57°$$

Tangent is negative in the second and fourth quadrants.
Therefore, $\theta = 180 - 57 = 123°$ and $\theta = 360 - 57 = 303°$.

Part III

13. *Answer:* $x = 0, 4 \pm 3i$

$$x^3 - 8x^2 + 25x = 0$$
$$x(x^2 - 8x + 25) = 0$$

Therefore, $x = 0$ is one solution.
Use the quadratic formula to find the roots of the quadratic factor:

$$x = \frac{8 \pm \sqrt{-36}}{2}$$
$$x = 4 \pm 3i$$

14. $$\sum_{n=0}^{5} 3(2)^{n-1} = 3(2)^{-1} + 3(2)^{0} + 3(2)^{1} + 3(2)^{2} + 3(2)^{3} + 3(2)^{4}$$
$$= 94.5$$

Part IV

15. **a.** Since $\overline{BG}$ is the diagonal of a square, $m\angle GBC = 45$.

b. $\tan\theta = \frac{GC}{AC} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

$\therefore\ \theta = \tan^{-1}\left(\frac{\sqrt{2}}{2}\right) \approx 35°$

16. *Answer:* $\left(\frac{1}{2}, -6\right)$ and $(2, -3)$

Substitute the linear equation into the quadratic and solve for x:

$$y = 2x - 7$$
$$2x - 7 = 2x^2 - 3x - 5$$
$$0 = 2x^2 - 5x + 2$$
$$0 = (2x - 1)(x - 2)$$
$$x = \tfrac{1}{2}, 2$$
$$y = 2\left(\tfrac{1}{2}\right) - 7 = -6$$
$$y = 2(2) - 7 = -3$$

Chapter 12. Trigonometric Identities

12-1 Basic Identities *(page 485)*

Writing About Mathematics

1. No. We also need to know the quadrant in which the terminal side of the angle lies to determine the sign of the other trigonometric functions.
2. **a.** To derive $1 + \tan^2\theta = \sec^2\theta$, divide each term of $\sin^2\theta + \cos^2\theta = 1$ by $\cos^2\theta$. To derive $\cot^2\theta + 1 = \csc^2\theta$, divide each term of $\sin^2\theta + \cos^2\theta = 1$ by $\sin^2\theta$.

   b. No; $\tan\theta$ and $\sec\theta$ are not defined for $\theta = \frac{\pi}{2} + n\pi$ and $\cot\theta$ and $\csc\theta$ are undefined for $\theta = n\pi$, so the identities are not defined for those values of θ.

Developing Skills

3. $\frac{\sin\theta}{\cos\theta}$	4. $\frac{\cos\theta}{\sin\theta}$	5. $\frac{1}{\cos\theta}$
6. $\frac{1}{\sin\theta}$	7. $\frac{1}{\sin\theta}$	8. $\frac{1}{\cos^2\theta}$
9. $\frac{1}{\sin^2\theta}$	10. $\frac{1}{\cos\theta}$	11. $\sin\theta\cos\theta$
12. $\frac{1}{\cos^2\theta}$	13. $\frac{2\cos\theta}{\sin\theta}$	14. $\frac{1 + \sin\theta\cos\theta}{\cos\theta}$

12-2 Proving an Identity *(pages 487–488)*

Writing About Mathematics

1. No. The equation is conditional. If θ is an angle whose terminal side lies in quadrant III or IV, then the equation is false.
2. Yes. The fraction is equal to 1. When the left side is multiplied and simplified it becomes $\frac{\cos^2\theta}{1 + \sin\theta}$.

Developing Skills

3. $\sin\theta\csc\theta\cos\theta \stackrel{?}{=} \cos\theta$

$$\frac{\cancel{\sin\theta}^{\,1}}{1}\cdot\frac{1}{\cancel{\sin\theta}_{\,1}}\cdot\frac{\cos\theta}{1} \stackrel{?}{=} \cos\theta$$

$$\cos\theta = \cos\theta\ ✔$$

4. $\tan\theta\sin\theta\cos\theta \stackrel{?}{=} \sin^2\theta$

$$\frac{\sin\theta}{\cancel{\cos\theta}_{\,1}}\cdot\frac{\sin\theta}{1}\cdot\frac{\cancel{\cos\theta}^{\,1}}{1} \stackrel{?}{=} \sin^2\theta$$

$$\sin^2\theta = \sin^2\theta\ ✔$$

5. $\cot\theta\sin\theta\cos\theta \stackrel{?}{=} \cos^2\theta$

$$\frac{\cos\theta}{\cancel{\sin\theta}_{\,1}}\cdot\frac{\cancel{\sin\theta}^{\,1}}{1}\cdot\frac{\cos\theta}{1} \stackrel{?}{=} \cos^2\theta$$

$$\cos^2\theta = \cos^2\theta\ ✔$$

6.
$$\sec\theta\,(\cos\theta-\cot\theta)\stackrel{?}{=}1-\csc\theta$$
$$\sec\theta\cos\theta-\sec\theta\cot\theta\stackrel{?}{=}1-\csc\theta$$
$$\frac{1}{\cancel{\cos\theta}}\cdot\frac{\cancel{\cos\theta}}{1}-\frac{1}{\cancel{\cos\theta}}\cdot\frac{\cancel{\cos\theta}}{\sin\theta}\stackrel{?}{=}1-\csc\theta$$
$$1-\frac{1}{\sin\theta}\stackrel{?}{=}1-\csc\theta$$
$$1-\csc\theta=1-\csc\theta\ \checkmark$$

7.
$$\csc\theta\,(\sin\theta+\tan\theta)\stackrel{?}{=}1+\sec\theta$$
$$\csc\theta\sin\theta+\csc\theta\tan\theta\stackrel{?}{=}1+\sec\theta$$
$$\frac{1}{\cancel{\sin\theta}}\cdot\frac{\cancel{\sin\theta}}{1}+\frac{1}{\cancel{\sin\theta}}\cdot\frac{\cancel{\sin\theta}}{\cos\theta}\stackrel{?}{=}1+\sec\theta$$
$$1+\frac{1}{\cos\theta}\stackrel{?}{=}1+\sec\theta$$
$$1+\sec\theta=1+\sec\theta\ \checkmark$$

8.
$$1-\frac{\cos\theta}{\sec\theta}\stackrel{?}{=}\sin^2\theta$$
$$1-\frac{\cos\theta}{\frac{1}{\cos\theta}}\stackrel{?}{=}\sin^2\theta$$
$$1-\cos^2\theta\stackrel{?}{=}\sin^2\theta$$
$$\sin^2\theta=\sin^2\theta\ \checkmark$$

9.
$$1-\frac{\sin\theta}{\csc\theta}\stackrel{?}{=}\cos^2\theta$$
$$1-\frac{\sin\theta}{\frac{1}{\sin\theta}}\stackrel{?}{=}\cos^2\theta$$
$$1-\sin^2\theta\stackrel{?}{=}\cos^2\theta$$
$$\cos^2\theta=\cos^2\theta\ \checkmark$$

10.
$$\sin\theta\,(\csc\theta-\sin\theta)\stackrel{?}{=}\cos^2\theta$$
$$\sin\theta\csc\theta-\sin^2\theta\stackrel{?}{=}\cos^2\theta$$
$$\frac{\cancel{\sin\theta}}{1}\cdot\frac{1}{\cancel{\sin\theta}}-\sin^2\theta\stackrel{?}{=}\cos^2\theta$$
$$1-\sin^2\theta\stackrel{?}{=}\cos^2\theta$$
$$\cos^2\theta=\cos^2\theta\ \checkmark$$

11.
$$\cos\theta\,(\sec\theta-\cos\theta)\stackrel{?}{=}\sin^2\theta$$
$$\cos\theta\sec\theta-\cos^2\theta\stackrel{?}{=}\sin^2\theta$$
$$\frac{\cancel{\cos\theta}}{1}\cdot\frac{1}{\cancel{\cos\theta}}-\cos^2\theta\stackrel{?}{=}\sin^2\theta$$
$$1-\cos^2\theta\stackrel{?}{=}\sin^2\theta$$
$$\sin^2\theta=\sin^2\theta\ \checkmark$$

12.
$$\frac{\tan\theta}{\sec\theta}\stackrel{?}{=}\sin\theta$$
$$\frac{\frac{\sin\theta}{\cos\theta}}{\frac{1}{\cos\theta}}\stackrel{?}{=}\sin\theta$$
$$\sin\theta=\sin\theta\ \checkmark$$

13.
$$\frac{\cot\theta}{\csc\theta}\stackrel{?}{=}\cos\theta$$
$$\frac{\frac{\cos\theta}{\sin\theta}}{\frac{1}{\sin\theta}}\stackrel{?}{=}\cos\theta$$
$$\cos\theta=\cos\theta\ \checkmark$$

14.
$$\frac{\csc\theta}{\sec\theta}\stackrel{?}{=}\cot\theta$$
$$\frac{\frac{1}{\sin\theta}}{\frac{1}{\cos\theta}}\stackrel{?}{=}\cot\theta$$
$$\frac{\cos\theta}{\sin\theta}\stackrel{?}{=}\cot\theta$$
$$\cot\theta=\cot\theta\ \checkmark$$

15.
$$\frac{\sec\theta}{\csc\theta}\stackrel{?}{=}\tan\theta$$
$$\frac{\frac{1}{\cos\theta}}{\frac{1}{\sin\theta}}\stackrel{?}{=}\tan\theta$$
$$\frac{\sin\theta}{\cos\theta}\stackrel{?}{=}\tan\theta$$
$$\tan\theta=\tan\theta\ \checkmark$$

16.
$$\frac{1}{\sin\theta\cos\theta}-\frac{\cos\theta}{\sin\theta}\stackrel{?}{=}\tan\theta$$
$$\frac{1}{\sin\theta\cos\theta}-\frac{\cos^2\theta}{\sin\theta\cos\theta}\stackrel{?}{=}\tan\theta$$
$$\frac{1-\cos^2\theta}{\sin\theta\cos\theta}\stackrel{?}{=}\tan\theta$$
$$\frac{\sin^2\theta}{\sin\theta\cos\theta}\stackrel{?}{=}\tan\theta$$
$$\frac{\sin\theta}{\cos\theta}\stackrel{?}{=}\tan\theta$$
$$\tan\theta=\tan\theta\ \checkmark$$

17.
$$\frac{1}{\sin\theta\cos\theta}-\frac{\sin\theta}{\cos\theta}\stackrel{?}{=}\cot\theta$$
$$\frac{1}{\sin\theta\cos\theta}-\frac{\sin^2\theta}{\sin\theta\cos\theta}\stackrel{?}{=}\cot\theta$$
$$\frac{1-\sin^2\theta}{\sin\theta\cos\theta}\stackrel{?}{=}\cot\theta$$
$$\frac{\cos^2\theta}{\sin\theta\cos\theta}\stackrel{?}{=}\cot\theta$$
$$\frac{\cos\theta}{\sin\theta}\stackrel{?}{=}\cot\theta$$
$$\cot\theta\stackrel{?}{=}\cot\theta\ \checkmark$$

18.
$$\frac{\sin^2\theta}{1+\cos\theta}\stackrel{?}{=}1-\cos\theta$$
$$\frac{\sin^2\theta}{1+\cos\theta}\cdot\frac{1-\cos\theta}{1-\cos\theta}\stackrel{?}{=}1-\cos\theta$$
$$\frac{\sin^2\theta\,(1-\cos\theta)}{1-\cos^2\theta}\stackrel{?}{=}1-\cos\theta$$
$$\frac{\cancel{\sin^2\theta}\,(1-\cos\theta)}{\cancel{\sin^2\theta}}\stackrel{?}{=}1-\cos\theta$$
$$1-\cos\theta=1-\cos\theta\ \checkmark$$

19.
$$\frac{\cos^2\theta}{1+\sin\theta}\stackrel{?}{=}1-\sin\theta$$
$$\frac{\cos^2\theta}{1+\sin\theta}\cdot\frac{1-\sin\theta}{1-\sin\theta}\stackrel{?}{=}1-\sin\theta$$
$$\frac{\cos^2\theta\,(1-\sin\theta)}{1-\sin^2\theta}\stackrel{?}{=}1-\sin\theta$$
$$\frac{\cancel{\cos^2\theta}\,(1-\sin\theta)}{\cancel{\cos^2\theta}}\stackrel{?}{=}1-\sin\theta$$
$$1-\sin\theta=1-\sin\theta\ \checkmark$$

20.
$$\sec\theta\csc\theta\stackrel{?}{=}\tan\theta+\cot\theta$$
$$\sec\theta\csc\theta\stackrel{?}{=}\frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{\sin\theta}$$
$$\sec\theta\csc\theta\stackrel{?}{=}\frac{\sin^2\theta}{\cos\theta\sin\theta}+\frac{\cos^2\theta}{\sin\theta\cos\theta}$$
$$\sec\theta\csc\theta\stackrel{?}{=}\frac{\sin^2\theta+\cos^2\theta}{\cos\theta\sin\theta}$$
$$\sec\theta\csc\theta\stackrel{?}{=}\frac{1}{\cos\theta\sin\theta}$$
$$\sec\theta\csc\theta=\sec\theta\csc\theta\ \checkmark$$

21.
$$\frac{\tan^2\theta}{\sec\theta-1}-1\stackrel{?}{=}\sec\theta$$
$$\frac{\sec^2\theta-1}{\sec\theta-1}-1\stackrel{?}{=}\sec\theta$$
$$\frac{(\sec\theta+1)\cancel{(\sec\theta-1)}}{\cancel{\sec\theta-1}}-1\stackrel{?}{=}\sec\theta$$
$$\sec\theta+1-1\stackrel{?}{=}\sec\theta$$
$$\sec\theta=\sec\theta\ \checkmark$$

22. $$\cos\theta + \frac{\sin^2\theta}{1+\cos\theta} \stackrel{?}{=} 1$$
$$\frac{\cos\theta}{1}\cdot\frac{1+\cos\theta}{1+\cos\theta} + \frac{\sin^2\theta}{1+\cos\theta} \stackrel{?}{=} 1$$
$$\frac{\cos\theta+\cos^2\theta}{1+\cos\theta} + \frac{\sin^2\theta}{1+\cos\theta} \stackrel{?}{=} 1$$
$$\frac{\cos\theta+\cos^2\theta+\sin^2\theta}{1+\cos\theta} \stackrel{?}{=} 1$$
$$\frac{\cos\theta+1}{1+\cos\theta} \stackrel{?}{=} 1$$
$$1 = 1\ ✔$$

23. $$\sin\theta + \frac{\cos^2\theta}{1+\sin\theta} \stackrel{?}{=} 1$$
$$\frac{\sin\theta}{1}\cdot\frac{1+\sin\theta}{1+\sin\theta} + \frac{\cos^2\theta}{1+\sin\theta} \stackrel{?}{=} 1$$
$$\frac{\sin\theta+\sin^2\theta}{1+\sin\theta} + \frac{\cos^2\theta}{1+\sin\theta} \stackrel{?}{=} 1$$
$$\frac{\sin\theta+\sin^2\theta+\cos^2\theta}{1+\sin\theta} \stackrel{?}{=} 1$$
$$\frac{\sin\theta+1}{1+\sin\theta} \stackrel{?}{=} 1$$
$$1 = 1\ ✔$$

24. $$\frac{\sec\theta}{\cos\theta} - \tan^2\theta \stackrel{?}{=} 1$$
$$\frac{\frac{1}{\cos\theta}}{\cos\theta} - \frac{\sin^2\theta}{\cos^2\theta} \stackrel{?}{=} 1$$
$$\frac{1}{\cos^2\theta} - \frac{\sin^2\theta}{\cos^2\theta} \stackrel{?}{=} 1$$
$$\frac{1-\sin^2\theta}{\cos^2\theta} \stackrel{?}{=} 1$$
$$\frac{\cos^2\theta}{\cos^2\theta} \stackrel{?}{=} 1$$
$$1 = 1\ ✔$$

25. $$\frac{\csc\theta}{\sin\theta} - \cot^2\theta \stackrel{?}{=} 1$$
$$\frac{\frac{1}{\sin\theta}}{\sin\theta} - \frac{\cos^2\theta}{\sin^2\theta} \stackrel{?}{=} 1$$
$$\frac{1}{\sin^2\theta} - \frac{\cos^2\theta}{\sin^2\theta} \stackrel{?}{=} 1$$
$$\frac{1-\cos^2\theta}{\sin^2\theta} \stackrel{?}{=} 1$$
$$\frac{\sin^2\theta}{\sin^2\theta} \stackrel{?}{=} 1$$
$$1 = 1\ ✔$$

26. $$\frac{\cos\theta}{\sec\theta} + \frac{\sin\theta}{\csc\theta} \stackrel{?}{=} 1$$
$$\frac{\cos\theta}{\frac{1}{\cos\theta}} + \frac{\sin\theta}{\frac{1}{\sin\theta}} \stackrel{?}{=} 1$$
$$\cos^2\theta + \sin^2\theta \stackrel{?}{=} 1$$
$$1 = 1\ ✔$$

27. It is undefined when sec θ or csc θ are undefined; that is, at $\frac{n\pi}{2}$ where n is an integer.

12-3 Cosine $(A - B)$ *(pages 491–493)*

Writing About Mathematics

1. Yes, the equations were shown to be true for all real numbers.
2. Yes. She used the identity $\cos(90 - B) = \sin B$ and she let $\angle B = 100°$.

Developing Skills

3. $-\frac{1}{2}$ 4. $-\frac{\sqrt{2}}{2}$ 5. $-\frac{\sqrt{3}}{2}$
6. $-\frac{\sqrt{3}}{2}$ 7. $-\frac{1}{2}$ 8. $\frac{\sqrt{3}}{2}$
9. $\frac{1}{2}$ 10. $\frac{\sqrt{3}}{2}$ 11. $-\frac{\sqrt{3}}{2}$
12. $-\frac{\sqrt{2}}{2}$ 13. $-\frac{1}{2}$ 14. $\frac{1}{2}$
15. $\frac{1}{2}$ 16. $-\frac{\sqrt{3}}{2}$ 17. -1

Applying Skills

18. **a.** $\frac{\sqrt{6}+\sqrt{2}}{4}$ **b.** $-\frac{\sqrt{6}+\sqrt{2}}{4}$
c. $\frac{\sqrt{6}+\sqrt{2}}{4}$ **d.** $\frac{\sqrt{6}+\sqrt{2}}{4}$

19. **a.** $-\frac{1}{2}$ **b.** $\frac{\sqrt{3}}{2}$
c. $\frac{\sqrt{6}-\sqrt{2}}{4}$ **d.** $\frac{\sqrt{2}-\sqrt{6}}{4}$
e. $\frac{\sqrt{6}-\sqrt{2}}{4}$ **f.** $\frac{\sqrt{6}-\sqrt{2}}{4}$

20. **a.** $-\frac{\sqrt{3}}{2}$ **b.** $-\frac{1}{2}$
c. $-\frac{\sqrt{6}+\sqrt{2}}{4}$ **d.** $\frac{\sqrt{6}+\sqrt{2}}{4}$
e. $-\frac{\sqrt{6}+\sqrt{2}}{4}$ **f.** $\frac{\sqrt{6}+\sqrt{2}}{4}$

21. **a.** 45°
b. $\sin\theta = \frac{4}{5} = 0.8$
$\cos\theta = \frac{3}{5} = 0.6$
c. $$\cos(\theta - 45°) = \cos\theta\cos 45° + \sin\theta\sin 45° = \left(\tfrac{3}{5}\right)\left(\tfrac{\sqrt{2}}{2}\right) + \left(\tfrac{4}{5}\right)\left(\tfrac{\sqrt{2}}{2}\right) = \frac{7\sqrt{2}}{10}$$
d. 8°

12-4 Cosine $(A + B)$ *(pages 495–496)*

Hands-On Activity

3. Draw segment $\overline{PP'}$ and let R be its point of intersection with the x-axis.
Then:
$$\sin\theta = RP \qquad \cos\theta = OR$$
$$\sin(-\theta) = -RP' \qquad \cos(-\theta) = OR$$
We have shown that $\cos\theta = \cos(-\theta)$. Since $RP = RP'$, $\sin\theta = -\sin(-\theta)$.

In steps 4–6, the procedures will be similar.

Writing About Mathematics

1. No. Maggie added the angles, which is incorrect. The correct answer is:
$$\begin{aligned}&\cos(A + B) + \cos(A - B)\\ &= \cos A\cos B - \sin A\sin B\\ &\quad + \cos A\cos(-B) - \sin A\sin(-B)\\ &= 2\cos A\cos B\end{aligned}$$

2. Yes. See the answer to Exercise 1.

Developing Skills

3. $-\frac{\sqrt{3}}{2}$ **4.** $-\frac{\sqrt{2}}{2}$ **5.** $-\frac{1}{2}$

6. $-\frac{1}{2}$ **7.** $-\frac{\sqrt{3}}{2}$ **8.** $-\frac{\sqrt{2}}{2}$

9. $\frac{1}{2}$ **10.** $\frac{\sqrt{3}}{2}$ **11.** $\frac{\sqrt{2}}{2}$

12. $-\frac{1}{2}$ **13.** $\frac{\sqrt{3}}{2}$ **14.** $\frac{\sqrt{2}}{2}$

15. $-\frac{\sqrt{3}}{2}$ **16.** $\frac{1}{2}$ **17.** $\frac{\sqrt{6}-\sqrt{2}}{4}$

Applying Skills

18. a. $\frac{\sqrt{6}-\sqrt{2}}{4}$ **b.** $\frac{\sqrt{2}-\sqrt{6}}{4}$ **c.** $\frac{\sqrt{6}-\sqrt{2}}{4}$

19. a. $-\frac{1}{2}$ **b.** $\frac{\sqrt{3}}{2}$

c. $-\frac{\sqrt{2}+\sqrt{6}}{4}$ **d.** $\frac{\sqrt{2}+\sqrt{6}}{4}$

20. a. $\frac{\sqrt{2}}{2}$ **b.** $-\frac{\sqrt{2}}{2}$ **c.** $\frac{\sqrt{6}+\sqrt{2}}{4}$

d. $$\begin{aligned}\cos 405° &= \cos(360° + 45°)\\ &= \cos 360° \cos 45° - \sin 360° \sin 45°\\ &= (1)(\cos 45°) - (0)(\sin 45°)\\ &= \cos 45°\end{aligned}$$

21. a. $AB = 50, \sin\theta = \frac{3}{5} = 0.6, \cos\theta = \frac{4}{5} = 0.8$

b. $$\begin{aligned}\cos(\theta + 45°) &= \cos\theta\cos 45° - \sin\theta\sin 45°\\ &= \left(\tfrac{4}{5}\right)\left(\tfrac{\sqrt{2}}{2}\right) - \left(\tfrac{3}{5}\right)\left(\tfrac{\sqrt{2}}{2}\right)\\ &= \tfrac{\sqrt{2}}{10}\end{aligned}$$

c. $$\begin{aligned}\cos(\theta + 45°) &= \tfrac{AC}{AD}\\ \tfrac{\sqrt{2}}{10} &= \tfrac{40}{AD}\\ AD &= 200\sqrt{2} \approx 282.84 \text{ ft}\end{aligned}$$

d. 280 ft

12-5 Sine (*A* − *B*) and Sine (*A* + *B*) *(pages 498–500)*

Writing About Mathematics

1. No. William added the angles, which is incorrect. The correct answer is:

$$\begin{aligned}&\sin(A+B) + \sin(A-B)\\ &= \sin A\cos B - \cos A\sin B\\ &\quad + \sin A\cos(-B) - \cos A\sin(-B)\\ &= 2\sin A\cos B\end{aligned}$$

2. Yes. See the answer to Exercise 1.

Developing Skills

3. $\sin(180° + 60°) = -\frac{\sqrt{3}}{2}$

$\sin(180° - 60°) = \frac{\sqrt{3}}{2}$

4. $\sin(180° + 45°) = -\frac{\sqrt{2}}{2}$

$\sin(180° - 45°) = \frac{\sqrt{2}}{2}$

5. $\sin(180° + 30°) = -\frac{1}{2}$

$\sin(180° - 30°) = \frac{1}{2}$

6. $\sin(270° + 60°) = -\frac{1}{2}$

$\sin(270° - 60°) = -\frac{1}{2}$

7. $\sin(270° + 30°) = -\frac{\sqrt{3}}{2}$

$\sin(270° - 30°) = -\frac{\sqrt{3}}{2}$

8. $\sin(60° + 90°) = \frac{1}{2}$

$\sin(60° - 90°) = -\frac{1}{2}$

9. $\sin(30° + 90°) = \frac{\sqrt{3}}{2}$

$\sin(30° - 90°) = -\frac{\sqrt{3}}{2}$

10. $\sin(90° + 60°) = \frac{1}{2}$

$\sin(90° - 60°) = \frac{1}{2}$

11. $\sin(60° + 270°) = -\frac{1}{2}$

$\sin(60° - 270°) = \frac{1}{2}$

12. $\sin(45° + 270°) = -\frac{\sqrt{2}}{2}$

$\sin(45° - 270°) = \frac{\sqrt{2}}{2}$

13. $\sin(30° + 270°) = -\frac{\sqrt{3}}{2}$

$\sin(30° - 270°) = \frac{\sqrt{3}}{2}$

14. $\sin(360° + 60°) = \frac{\sqrt{3}}{2}$

$\sin(360° - 60°) = -\frac{\sqrt{3}}{2}$

15. $\sin\left(\frac{3\pi}{2} + 2\pi\right) = -1$

$\sin\left(\frac{3\pi}{2} - 2\pi\right) = -1$

16. $\sin\left(\frac{2\pi}{3} + \frac{\pi}{6}\right) = \frac{1}{2}$

$\sin\left(\frac{2\pi}{3} - \frac{\pi}{6}\right) = 1$

17. $\sin\left(\frac{\pi}{3} + \frac{5\pi}{4}\right) = -\frac{\sqrt{6}+\sqrt{2}}{4}$

$\sin\left(\frac{\pi}{3} - \frac{5\pi}{4}\right) = \frac{\sqrt{2}-\sqrt{6}}{4}$

Applying Skills

18. a. $\frac{\sqrt{6}-\sqrt{2}}{4}$ **b.** $\frac{\sqrt{6}-\sqrt{2}}{4}$

c. $\frac{\sqrt{2}-\sqrt{6}}{4}$ **d.** $\frac{\sqrt{2}-\sqrt{6}}{4}$

19. a. $\frac{\sqrt{3}}{2}$ **b.** $-\frac{1}{2}$

c. $\frac{\sqrt{6}+\sqrt{2}}{4}$ **d.** $\frac{\sqrt{6}+\sqrt{2}}{4}$

e. $-\frac{\sqrt{6}+\sqrt{2}}{4}$

20. a. $-\frac{1}{2}$ **b.** $-\frac{\sqrt{3}}{2}$

c. $\frac{\sqrt{6}-\sqrt{2}}{4}$ **d.** $\frac{\sqrt{2}-\sqrt{6}}{4}$

e. $\frac{\sqrt{2}-\sqrt{6}}{4}$ **f.** $\frac{\sqrt{6}+\sqrt{2}}{4}$

21. a. $\sin x = \frac{17}{33}$

$\cos x = \frac{20\sqrt{2}}{33}$

$\sin y = \frac{1}{3}$

$\cos y = \frac{2\sqrt{2}}{3}$

b. $\sin(x - y) = \sin x \cos y - \cos x \sin y$

$= \left(\frac{17}{33}\right)\left(\frac{2\sqrt{2}}{3}\right) - \left(\frac{20\sqrt{2}}{33}\right)\left(\frac{1}{3}\right)$

$= \frac{14}{99}\sqrt{2}$

c. 12°

22. a. $\sin\theta = \frac{5}{13}, \cos\theta = \frac{12}{13}$

b. $\sin(\theta + 30°) = \sin\theta\cos 30° + \cos\theta\sin 30°$

$= \left(\frac{5}{13}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{12}{13}\right)\left(\frac{1}{2}\right)$

$= \frac{5\sqrt{3} + 12}{26}$

c. $\frac{5\sqrt{3} + 12}{26} = \frac{500}{x}$

$x = \frac{13{,}000}{5\sqrt{3} + 12}$

$x \approx 629.23$ ft

23. *Answer:* $A'(-\sqrt{2}, 7\sqrt{2})$

$r = \sqrt{6^2 + 8^2} = 10, \quad \cos a = \frac{3}{5}, \quad \sin a = \frac{4}{5}$

Therefore, $A' = (10\cos(a + 45), 10\sin(a + 45))$.

$10\cos(a + 45°) = 10(\cos a\cos 45° - \sin a\sin 45°)$

$= 10\left[\left(\frac{3}{5}\right)\left(\frac{\sqrt{2}}{2}\right) - \left(\frac{4}{5}\right)\left(\frac{\sqrt{2}}{2}\right)\right]$

$= 10\left(-\frac{\sqrt{2}}{10}\right) = -\sqrt{2}$

$10\sin(a + 45°) = 10(\sin a\cos 45° + \cos a\sin 45°)$

$= 10\left[\left(\frac{4}{5}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(\frac{3}{5}\right)\left(\frac{\sqrt{2}}{2}\right)\right]$

$= 10\left(\frac{7\sqrt{2}}{10}\right) = 7\sqrt{2}$

12-6 Tangent ($A - B$) and Tangent ($A + B$) *(pages 502–504)*

Writing About Mathematics

1. If A or B is equal to $\frac{\pi}{2} + n\pi$ for any integer n, then tan A or tan B is undefined.
2. When $A = \frac{\pi}{6}$ and $B = \frac{\pi}{3}$, then $\tan A \tan B = 1$. That makes the denominator of the fraction zero and the fraction undefined.

Developing Skills

3. $\tan(45° + 30°) = 2 + \sqrt{3}$
$\tan(45° - 30°) = 2 - \sqrt{3}$

4. $\tan(45° + 60°) = -2 - \sqrt{3}$
$\tan(45° - 60°) = -2 + \sqrt{3}$

5. $\tan(60° + 60°) = -\sqrt{3}$
$\tan(60° - 60°) = 0$

6. $\tan(180° + 30°) = \frac{\sqrt{3}}{3}$
$\tan(180° - 30°) = -\frac{\sqrt{3}}{3}$

7. $\tan(180° + 45°) = 1$
$\tan(180° - 45°) = -1$

8. $\tan(180° + 60°) = \sqrt{3}$
$\tan(180° - 60°) = -\sqrt{3}$

9. $\tan(120° + 30°) = -\frac{\sqrt{3}}{3}$
$\tan(120° - 30°) =$ undefined

10. $\tan(120° + 45°) = \sqrt{3} - 2$
$\tan(120° - 45°) = 2 + \sqrt{3}$

11. $\tan(120° + 60°) = 0$
$\tan(120° - 60°) = \sqrt{3}$

12. $\tan(120° + 120°) = \sqrt{3}$
$\tan(120° - 120°) = 0$

13. $\tan(240° + 120°) = 0$
$\tan(240° - 120°) = -\sqrt{3}$

14. $\tan(360° + 60°) = \sqrt{3}$
$\tan(360° - 60°) = -\sqrt{3}$

15. $\tan\left(\pi + \frac{\pi}{3}\right) = \sqrt{3}$
$\tan\left(\pi - \frac{\pi}{3}\right) = -\sqrt{3}$

16. $\tan\left(\frac{5\pi}{6} + \frac{5\pi}{6}\right) = -\sqrt{3}$
$\tan\left(\frac{5\pi}{6} - \frac{5\pi}{6}\right) = 0$

17. $\tan\left(\frac{\pi}{3} + \frac{\pi}{4}\right) = -2 - \sqrt{3}$
$\tan\left(\frac{\pi}{3} - \frac{\pi}{4}\right) = 2 - \sqrt{3}$

Applying Skills

18. $\tan(180 + \theta) = \frac{\tan 180° + \tan\theta}{1 - \tan 180°\tan\theta}$

$= \frac{0 + \tan\theta}{1 - (0)(\tan\theta)}$

$= \tan\theta$

19. 1 **20.** $-\frac{7}{4}$

21. $\cos A = -\sqrt{1 - (0.6)^2} = -0.8$

$\tan A = \frac{\sin A}{\cos A} = -0.75$

$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$

$\tan(A + B) = \frac{0.75 + 4}{1 - (0.75)(4)}$

$\tan(A + B) = -2.375$

22. $\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A\tan B}$

$\tan(A - B) = \frac{2 - (-2)}{1 + (2)(-2)}$

$\tan(A - B) = -\frac{4}{3}$

23. $\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$

$\tan(A + B) = \frac{-\frac{2}{3} + \frac{2}{3}}{1 - \left(-\frac{2}{3}\right)\left(\frac{2}{3}\right)}$

$\tan(A + B) = 0$

24. a. 1

b. Yes, $\tan(x + y) = 1$ and $\tan z = \frac{1}{1} = 1$.

25. a. $\frac{4}{19}$ **b.** 12°

26. a. $\tan x = \frac{4}{5}, \tan y = \frac{2}{5}$ **b.** $\frac{10}{33}$

12-7 Functions of 2*A* *(pages 507–508)*

Writing About Mathematics

1. No. Let $2\theta = A$. Using the cofunction identity, $\cos A = \sin(90° - A)$. Then by substitution, $\cos 2\theta = \sin(90° - 2\theta)$.
2. Yes. Let $2\theta = A$. Using the quotient identity, $\tan A = \frac{\sin A}{\cos A}$. Then by substitution, $\tan 2\theta = \frac{\sin 2\theta}{\cos 2\theta}$.

Developing Skills

3. a. $\frac{\sqrt{3}}{2}$ **b.** $\frac{1}{2}$ **c.** $\sqrt{3}$

4. a. 1 **b.** 0 **c.** Undefined

5. a. $-\frac{\sqrt{3}}{2}$ **b.** $\frac{1}{2}$ **c.** $-\sqrt{3}$

6. a. 1 **b.** 0 **c.** Undefined

7. a. $\frac{\sqrt{3}}{2}$ **b.** $\frac{1}{2}$ **c.** $\sqrt{3}$

8. a. $-\frac{\sqrt{3}}{2}$ **b.** $-\frac{1}{2}$ **c.** $\sqrt{3}$

9. a. $\frac{15}{17}$ **b.** $\frac{8}{17}$ **c.** $\frac{15}{8}$ **d.** Quadrant I

10. a. $-\frac{5\sqrt{11}}{18}$ **b.** $\frac{7}{18}$ **c.** $-\frac{5\sqrt{11}}{7}$ **d.** Quadrant IV

11. a. $\frac{20\sqrt{6}}{49}$ **b.** $\frac{1}{49}$ **c.** $20\sqrt{6}$ **d.** Quadrant I

12. a. $\frac{\sqrt{3}}{2}$ **b.** $\frac{1}{2}$ **c.** $\sqrt{3}$ **d.** Quadrant I

13. a. $\frac{12\sqrt{10}}{49}$ **b.** $\frac{31}{49}$ **c.** $\frac{12\sqrt{10}}{31}$ **d.** Quadrant I

14. a. $-\frac{4}{5}$ **b.** $\frac{3}{5}$ **c.** $-\frac{4}{3}$ **d.** Quadrant IV

15. a. $\frac{120}{169}$ **b.** $-\frac{119}{169}$ **c.** $-\frac{120}{119}$ **d.** Quadrant II

16. a. $\frac{2\sqrt{14}}{9}$ **b.** $\frac{5}{9}$ **c.** $\frac{2\sqrt{14}}{5}$ **d.** Quadrant I

17. a. $-\frac{4}{5}$ **b.** $\frac{3}{5}$ **c.** $-\frac{4}{3}$ **d.** Quadrant IV

18. a. $-\frac{12}{13}$ **b.** $-\frac{5}{13}$ **c.** $\frac{12}{5}$ **d.** Quadrant III

19. a. $-\frac{3}{5}$ **b.** $-\frac{4}{5}$ **c.** $\frac{3}{4}$ **d.** Quadrant III

20. a. $\frac{4}{5}$ **b.** $-\frac{3}{5}$ **c.** $-\frac{4}{3}$ **d.** Quadrant II

21. $\cot\theta \stackrel{?}{=} \frac{\sin 2\theta}{1 - \cos 2\theta}$

$\cot\theta \stackrel{?}{=} \frac{2\sin\theta\cos\theta}{1 - (1 - 2\sin^2\theta)}$

$\cot\theta \stackrel{?}{=} \frac{2\sin\theta\cos\theta}{2\sin^2\theta}$

$\cot\theta \stackrel{?}{=} \frac{\cos\theta}{\sin\theta}$

$\cot\theta = \cot\theta$ ✔

22. $\frac{\cos 2\theta}{\sin\theta} + \frac{\sin 2\theta}{\cos\theta} \stackrel{?}{=} \csc\theta$

$\frac{\cos\theta(\cos 2\theta) + \sin\theta(\sin 2\theta)}{\sin\theta\cos\theta} \stackrel{?}{=} \csc\theta$

$\frac{\overset{1}{\cancel{\cos\theta}}(\cos^2\theta - \sin^2\theta) + \sin\theta(2\sin\theta\overset{1}{\cancel{\cos\theta}})}{\sin\theta\underset{1}{\cancel{\cos\theta}}} \stackrel{?}{=} \csc\theta$

$\frac{\cos^2\theta - \sin^2\theta + 2\sin^2\theta}{\sin\theta} \stackrel{?}{=} \csc\theta$

$\frac{\cos^2\theta + \sin^2\theta}{\sin\theta} \stackrel{?}{=} \csc\theta$

$\frac{1}{\sin\theta} \stackrel{?}{=} \csc\theta$

$\csc\theta = \csc\theta$ ✔

23. $\cos 2\theta = \cos^2\theta - \sin^2\theta$

$\cos 2\theta = \cos^2\theta + \sin^2\theta - \sin^2\theta - \sin^2\theta$

$\cos 2\theta = (\cos^2\theta + \sin^2\theta) - (\sin^2\theta + \sin^2\theta)$

$\cos 2\theta = 1 - 2\sin^2\theta$

$\cos 2\theta = \cos 2\theta$ ✔

24. $\csc 2\theta \stackrel{?}{=} \frac{1}{2}\sec\theta\csc\theta$

$\frac{1}{\sin 2\theta} \stackrel{?}{=} \frac{1}{2}\sec\theta\csc\theta$

$\frac{1}{2\sin\theta\cos\theta} \stackrel{?}{=} \frac{1}{2}\sec\theta\csc\theta$

$\frac{1}{2}\left(\frac{1}{\sin\theta}\right)\left(\frac{1}{\cos\theta}\right) \stackrel{?}{=} \frac{1}{2}\sec\theta\csc\theta$

$\frac{1}{2}\sec\theta\csc\theta = \frac{1}{2}\sec\theta\csc\theta$ ✔

25. a. Let $4A = 2\theta$, then $2A = \theta$.
$\sin 2\theta = 2\sin\theta\cos\theta$
$\sin 4A = 2\sin 2A\cos 2A$

b. Let $4A = 2\theta$, then $2A = \theta$.
$\cos 2\theta = \cos^2\theta - \sin^2\theta$
$\cos 4A = \cos^2 2A - \sin^2 2A$

c. Let $4A = 2\theta$, then $2A = \theta$.
$\tan 2\theta = \frac{2\tan\theta}{1 - \tan^2\theta}$
$\tan 4A = \frac{2\tan 2A}{1 - \tan^2 2A}$

26. a. $\frac{5}{6}$

b. $m\angle BAC = 2\theta = \frac{7}{18}$

c. $\frac{180}{7}$ mi or $25\frac{5}{7}$ mi

27. $x = r\cos\theta$, $y = r\sin\theta$

$\cos\theta = \frac{\sqrt{10}}{4}$, $\sin\theta = \frac{\sqrt{6}}{4}$, $r = 4$

$\cos 2\theta = \frac{10 - 6}{16} = \frac{1}{4}$

$\sin 2\theta = 2\left(\frac{\sqrt{6}}{4}\right)\left(\frac{\sqrt{10}}{4}\right) = \frac{\sqrt{15}}{4}$

$A''\left(1, \sqrt{15}\right)$

12-8 Functions of $\frac{1}{2}A$ *(pages 511–513)*

Writing About Mathematics

1. Yes. Cosine is positive in the first and fourth quadrants, that is, $\cos A > 0$ when $-\frac{\pi}{2} < A < \frac{\pi}{2}$. $\frac{1}{2}A$ is in the first or fourth quadrant since $-\frac{\pi}{4} < \frac{A}{2} < \frac{\pi}{4}$, so $\cos\frac{1}{2}A$ is positive.

2. Yes. Let $\frac{1}{2}A = \theta$. Using the quotient identity,

$\tan\theta = \frac{\sin\theta}{\cos\theta}$. Then by substitution,

$\tan\frac{1}{2}A = \frac{\sin\frac{1}{2}A}{\cos\frac{1}{2}A}$.

Developing Skills

	a.	b.	c.
3.	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$\sqrt{3}$
4.	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
5.	$\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{3}}{3}$
6.	0	-1	0
7.	$-\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	-1
8.	$\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{2}}{2}$	-1
9.	$\frac{\sqrt{2}}{4}$	$\frac{\sqrt{14}}{4}$	$\frac{\sqrt{7}}{7}$
10.	$\frac{\sqrt{102}}{12}$	$\frac{\sqrt{42}}{12}$	$\frac{\sqrt{119}}{7}$
11.	$\frac{\sqrt{5}}{3}$	$-\frac{2}{3}$	$-\frac{\sqrt{5}}{2}$
12.	$\frac{\sqrt{7}}{4}$	$-\frac{3}{4}$	$-\frac{\sqrt{7}}{3}$
13.	$-\frac{\sqrt{15}}{5}$	$-\frac{\sqrt{10}}{5}$	$\frac{\sqrt{6}}{2}$
14.	$-\frac{1}{3}$	$-\frac{2\sqrt{2}}{3}$	$\frac{\sqrt{2}}{4}$
15.	$\frac{3}{5}$	$\frac{4}{5}$	$\frac{3}{4}$
16.	$\frac{\sqrt{5}}{5}$	$-\frac{2\sqrt{5}}{5}$	$-\frac{1}{2}$
17.	$-\frac{3}{5}$	$\frac{4}{5}$	$-\frac{3}{4}$

18. a. $\frac{\sqrt{50 + 5\sqrt{10}}}{10}$

b. $-\frac{\sqrt{50 - 5\sqrt{10}}}{10}$

c. $-\frac{1 + \sqrt{10}}{3}$

Applying Skills

19.
$$\tan\frac{1}{2}A = \pm\sqrt{\frac{1 - \cos A}{1 + \cos A}} = \pm\sqrt{\frac{1 - \cos A}{1 + \cos A} \times \frac{1 - \cos A}{1 - \cos A}} = \pm\sqrt{\frac{(1 - \cos A)^2}{1 - \cos^2 A}} = \pm\sqrt{\frac{(1 - \cos A)^2}{\sin^2 A}} = \pm\frac{1 - \cos A}{\sin A}$$

20.
$$\tan\frac{1}{8} = \sqrt{\frac{1 - \frac{\sqrt{2}}{2}}{1 + \frac{\sqrt{2}}{2}}} = \sqrt{\frac{2 - \sqrt{2}}{2 + \sqrt{2}} \cdot \frac{2 - \sqrt{2}}{2 - \sqrt{2}}} = \sqrt{\frac{(2 - \sqrt{2})^2}{2}} = \frac{2 - \sqrt{2}}{\sqrt{2}} = \sqrt{2} - 1$$

21.
$$\tan 15° = \sqrt{\frac{1 - \frac{\sqrt{3}}{2}}{1 + \frac{\sqrt{3}}{2}}} = \sqrt{\frac{2 - \sqrt{3}}{2 + \sqrt{3}} \cdot \frac{2 - \sqrt{3}}{2 - \sqrt{3}}} = \sqrt{\frac{(2 - \sqrt{3})^2}{1}} = 2 - \sqrt{3}$$

22.
$$\sin 15° = \sqrt{\frac{1 - \frac{\sqrt{3}}{2}}{2}} = \sqrt{\frac{2 - \sqrt{3}}{4}} = \frac{\sqrt{2 - \sqrt{3}}}{2}$$

23. a. Let $\frac{1}{4}A = \frac{1}{2}\theta$, then $\frac{1}{2}A = \theta$.

$\sin\frac{1}{2}\theta = \pm\sqrt{\frac{1 - \cos\theta}{2}}$

$\sin\frac{1}{4}A = \pm\sqrt{\frac{1 - \cos\frac{1}{2}A}{2}}$

b. Let $\frac{1}{4}A = \frac{1}{2}\theta$, then $\frac{1}{2}A = \theta$.

$\cos\frac{1}{2}\theta = \pm\sqrt{\frac{1 + \cos\theta}{2}}$

$\cos\frac{1}{4}A = \pm\sqrt{\frac{1 + \cos\frac{1}{2}A}{2}}$

c. Let $\frac{1}{4}A = \frac{1}{2}\theta$, then $\frac{1}{2}A = \theta$.

$$\tan\tfrac{1}{2}\theta = \pm\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}$$

$$\tan\tfrac{1}{4}A = \pm\sqrt{\frac{1-\cos\frac{1}{2}A}{1+\cos\frac{1}{2}A}}$$

24. a. $\cos\theta = \frac{25}{65} = \frac{5}{13}$

$$\tan\tfrac{1}{2}\theta = \sqrt{\frac{1-\frac{5}{13}}{1+\frac{5}{13}}}$$

$$= \sqrt{\tfrac{4}{9}} = \tfrac{2}{3}$$

b. $\tan\frac{1}{2}\theta = \frac{\text{height of base}}{25}$

$$\tfrac{2}{3} = \tfrac{h}{25}$$

$$h = \tfrac{50}{3} \text{ or } 16\tfrac{2}{3} \text{ ft}$$

height of bill board $= 60 - \frac{50}{3} = 43\frac{1}{3}$ ft

Hands-On Activity: Graphical Support for the Trigonometric Identities

1. Yes
2. Yes
3. Yes
4. Each graph, Y_2 and Y_3, coincides with Y_1 only part of the time. When $\cos\frac{x}{2}$ is positive, Y_2 coincides and when $\cos\frac{x}{2}$ is negative, Y_3 coincides. Neither Y_2 nor Y_3 is accurate for all values of x.

Review Exercises *(page 514)*

1. $\sec\theta \stackrel{?}{=} \csc\theta\tan\theta$

$$\sec\theta \stackrel{?}{=} \frac{1}{\cancel{\sin\theta}}\cdot\frac{\cancel{\sin\theta}}{\cos\theta}$$

$$\sec\theta \stackrel{?}{=} \frac{1}{\cos\theta}$$

$$\sec\theta = \sec\theta ✔$$

2. $\cos\theta\cot\theta + \sin\theta \stackrel{?}{=} \csc\theta$

$$\cos\theta\frac{\cos\theta}{\sin\theta} + \sin\theta \stackrel{?}{=} \csc\theta$$

$$\frac{\cos^2\theta}{\sin\theta} + \sin\theta\left(\frac{\sin\theta}{\sin\theta}\right) \stackrel{?}{=} \csc\theta$$

$$\frac{\cos^2\theta + \sin^2\theta}{\sin\theta} \stackrel{?}{=} \csc\theta$$

$$\frac{1}{\sin\theta} \stackrel{?}{=} \csc\theta$$

$$\csc\theta = \csc\theta ✔$$

3. $2\sin^2\theta \stackrel{?}{=} 1 - \cos 2\theta$

$$2\sin^2\theta \stackrel{?}{=} 1 - (1 - 2\sin^2\theta)$$

$$2\sin^2\theta = 2\sin^2\theta ✔$$

4. $\tan\theta + \frac{1}{\csc\theta} \stackrel{?}{=} \frac{1+\cos\theta}{\cot\theta}$

$$\frac{\sin\theta}{\cos\theta} + \sin\theta \stackrel{?}{=} \frac{1+\cos\theta}{\cot\theta}$$

$$\frac{\sin\theta}{\cos\theta} + \frac{\sin\theta\cos\theta}{\cos\theta} \stackrel{?}{=} \frac{1+\cos\theta}{\cot\theta}$$

$$\frac{\sin\theta(1+\cos\theta)}{\cos\theta} \stackrel{?}{=} \frac{1+\cos\theta}{\cot\theta}$$

$$\tan\theta(1+\cos\theta) \stackrel{?}{=} \frac{1+\cos\theta}{\cot\theta}$$

$$\frac{1+\cos\theta}{\cot\theta} = \frac{1+\cos\theta}{\cot\theta} ✔$$

5. $\frac{\sin 2\theta + \sin\theta}{\cos 2\theta + \cos\theta + 1} \stackrel{?}{=} \tan\theta$

$$\frac{2\sin\theta\cos\theta + \sin\theta}{2\cos^2\theta - 1 + \cos\theta + 1} \stackrel{?}{=} \tan\theta$$

$$\frac{\sin\theta(2\cos\theta + 1)}{\cos\theta(2\cos\theta + 1)} \stackrel{?}{=} \tan\theta$$

$$\frac{\sin\theta}{\cos\theta} \stackrel{?}{=} \tan\theta$$

$$\tan\theta = \tan\theta ✔$$

6. $\frac{\sin(A+B) + \sin(A-B)}{\sin(A+B) - \sin(A-B)} \stackrel{?}{=} \tan A\cot B$

$$\frac{\sin A\cos B + \cancel{\cos A\sin B} + \sin A\cos B - \cancel{\cos A\sin B}}{\cancel{\sin A\cos B} + \cos A\sin B - \cancel{\sin A\cos B} + \cos A\sin B} \stackrel{?}{=} \tan A\cot B$$

$$\frac{2\sin A\cos B}{2\cos A\sin B} \stackrel{?}{=} \tan A\cot B$$

$$\frac{\sin A}{\cos A}\cdot\frac{\cos B}{\sin B} \stackrel{?}{=} \tan A\cot B$$

$$\tan A\cot B = \tan A\cot B ✔$$

7. $-\frac{24}{25}$	**8.** $-\frac{4}{5}$	**9.** $\frac{7}{24}$
10. $\frac{3}{4}$	**11.** $\frac{4}{5}$	**12.** $-\frac{44}{125}$
13. $\frac{3}{5}$	**14.** $\frac{117}{125}$	**15.** $\frac{4}{3}$
16. $-\frac{44}{117}$	**17.** $\frac{336}{625}$	**18.** $\frac{7}{25}$
19. $\frac{336}{527}$	**20.** $\frac{7\sqrt{2}}{10}$	**21.** $-\frac{\sqrt{2}}{10}$
22. $\frac{8\sqrt{6}}{5}$	**23.** -0.8	**24.** $\frac{3}{4}$
25. $\frac{\sqrt{7}}{3}$	**26.** $\frac{3\sqrt{7}}{8}$	**27.** $\frac{1}{8}$
28. $3\sqrt{7}$	**29.** $\frac{\sqrt{2}}{4}$	**30.** $\frac{\sqrt{14}}{4}$
31. $\frac{\sqrt{7}}{7}$	**32.** $-\frac{9}{16}$	**33.** $\frac{5\sqrt{2}}{7}$
34. $\frac{7\sqrt{2}}{10}$	**35.** $\frac{\sqrt{2}}{10}$	**36.** 7
37. $\frac{7}{25}$	**38.** $\frac{24}{25}$	**39.** $\frac{7}{24}$

40. $-\frac{\sqrt{50 - 35\sqrt{2}}}{10}$ **41.** $-\frac{\sqrt{50 + 35\sqrt{2}}}{10}$

42. If A and B complementary, $\cos(A+B) = \cos 90°$.

$$\cos(A+B) = \cos A\cos B - \sin A\sin B$$

$$\cos 90° = \cos A\cos B - \sin A\sin B$$

$$0 = \cos A\cos B - \sin A\sin B$$

$$\sin A\sin B = \cos A\cos B ✔$$

Exploration *(page 515)*

1. The equations appear to be identities since when each left side is graphed in Y_1 and each right side is graphed in Y_2, the graphs of Y_1 and Y_2 coincide.

2. $\sin(2A + A) = \sin 2A \cos A + \cos 2A \sin A$
$\sin(2A + A) = (2\sin A \cos A)\cos A + (2\cos^2 A - 1)\sin A$
$\sin(2A + A) = 2\sin A \cos^2 A + 2\sin A \cos^2 A - \sin A$
$\sin(2A + A) = \sin A\,(4\cos^2 A - 1)$
$\sin(2A + A) = \sin A\,(4(1 - \sin^2 A) - 1)$
$\sin(2A + A) = \sin A\,(3 - 4\sin^2 A)$
$\sin(3A) = 3\sin A - 4\sin^3 A$ ✔

$\cos(2A + A) = \cos 2A \cos A - \sin 2A \sin A$
$\cos(2A + A) = (2\cos^2 A - 1)\cos A - (2\sin A \cos A)\sin A$
$\cos(2A + A) = (2\cos^2 A - 1)\cos A - (2\sin^2 A)\cos A$
$\cos(2A + A) = \cos A\,[(2\cos^2 A - 1) - 2\sin^2 A]$
$\cos(2A + A) = \cos A\,[2\cos^2 A - 1 - 2(1 - \cos^2 A)]$
$\cos(2A + A) = \cos A\,(4\cos^2 A - 3)$
$\cos(3A) = 4\cos^3 A - 3\cos A$ ✔

$$\tan(2A + A) = \frac{\tan 2A + \tan A}{1 - \tan 2A \tan A}$$

$$\tan(2A + A) = \frac{\frac{2\tan A}{1 - \tan^2 A} + \tan A}{1 - \frac{2\tan A}{1 - \tan^2 A}\tan A}$$

$$\tan(2A + A) = \frac{\left(\frac{2\tan A}{1 - \tan^2 A} + \tan A\right)}{\left(1 - \frac{2\tan^2 A}{1 - \tan^2 A}\right)} \times \frac{1 - \tan^2 A}{1 - \tan^2 A}$$

$$\tan(2A + A) = \frac{2\tan A + \tan A(1 - \tan^2 A)}{1 - \tan^2 A - 2\tan^2 A}$$

$$\tan(3A) = \frac{3\tan A - \tan^3 A}{1 - 3\tan^2 A} ✔$$

Cumulative Review *(pages 515–517)*

Part I

1. 4	**2.** 1	**3.** 2
4. 3	**5.** 4	**6.** 1
7. 1	**8.** 2	**9.** 3
10. 3		

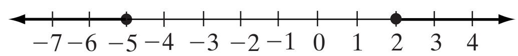
Part II

11. $\frac{1}{1+i} = \frac{1}{1+i} \cdot \frac{1-i}{1-i} = \frac{1-i}{2} = \frac{1}{2} - \frac{1}{2}i$

12. $a = \frac{5}{2}$ and $c = \frac{25}{4}$
If $x^2 + 5x + c = (x + a)^2$, then
$x^2 + 5x + c = x^2 + 2ax + a^2$.
Therefore, $2a = 5$, so $a = \frac{5}{2}$ and $a^2 = c = \frac{25}{4}$.

Part III

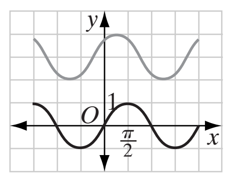
13. $x^2 + 3x - 10 \geq 10$
$(x + 5)(x - 2) \geq 0$
$x \leq -5$ or $x \geq 2$



14. center: $(1, 2)$
radius: $\sqrt{(2 - 1)^2 + (5 - 2)^2} = \sqrt{10}$
equation: $(x - 1)^2 + (y - 2)^2 = 10$

Part IV

15. a.



b. $g(x) = 3 + \sin\left(x + \frac{\pi}{4}\right)$

16. a. $a_5 = a_1 r^{5-1}$
$9 = r^4$
$r = \sqrt{3}$

b. $1, \sqrt{3}, 3, 3\sqrt{3}, 9, 9\sqrt{3}, 27, 27\sqrt{3}$

c. $\sum_{n=1}^{8} (\sqrt{3})^{n-1}$ or $\sum_{n=1}^{8} 3^{\frac{n-1}{2}}$

Chapter 13. Trigonometric Equations

13-1 First Degree Trigonometric Equations *(pages 524–526)*

Writing About Mathematics

1. The second equation simplifies to $\sin x = 2$ and 2 is outside the range of the sine function.

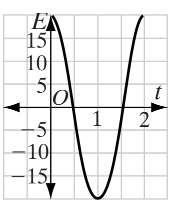
2. The second equation simplifies to $\tan x = 1$. Since the tangent function is periodic, there are an infinite number of x-values where $\tan x = 1$.

Developing Skills

3. 60°, 300°
4. 150°, 330°
5. 90°
6. 90°, 270°
7. 135°, 315°
8. 45°, 315°
9. $\frac{\pi}{3}, \frac{2\pi}{3}$
10. 0
11. $\frac{\pi}{4}, \frac{5\pi}{4}$
12. $\frac{5\pi}{4}, \frac{7\pi}{4}$
13. $\frac{\pi}{6}, \frac{5\pi}{6}$
14. $\frac{\pi}{2}, \frac{3\pi}{2}$
15. 49°
16. 79°
17. 71°
18. 24°
19. 12°
20. 18°
21. 75.5°, 284.5°
22. 16.6°, 163.4°
23. 104.0°, 284.0°
24. 131.8°, 228.2°
25. 0.17, 2.97
26. 2.09, 4.19 or $\frac{2\pi}{3}, \frac{4\pi}{3}$
27. 0.38, 3.52
28. 3.02, 6.16

Applying Skills

29. a.



b. −20 volts
c. 2
d. (1) $\theta = 0.93, 5.36$
(2) $t = 0.30$ s and 1.70 s

30. a. $x = 36.9°$
b. $\theta = 18.4°$

31. a. We used the expression $\frac{1}{\tan \theta}$ for the cotangent function, which is undefined at $\frac{\pi}{2}$.

b. Yes. $\cot\left(\frac{\pi}{2}\right) = \frac{\cos \frac{\pi}{2}}{\sin \frac{\pi}{2}} = \frac{0}{1} = 0$ and $\sin\left(\frac{\pi}{2} - \frac{\pi}{2}\right) = 0$.

13-2 Using Factoring to Solve Trigonometric Equations *(pages 529–530)*

Writing About Mathematics

1. No. The method of using factoring to solve a trigonometric equation depends on the multiplicative property of zero: If $ab = 0$, then $a = 0$ or $b = 0$. Thus, the right side of the equation must equal 0.

2. Yes. $2(\sin \theta)(\cos \theta) + \sin \theta + 2 \cos \theta + 1 = 0$

$$\sin \theta\,(2 \cos \theta + 1) + 1(2 \cos \theta + 1) = 0$$
$$(\sin \theta + 1)(2 \cos \theta + 1) = 0$$
$$\sin \theta + 1 = 0 \mid 2 \cos \theta + 1 = 0$$

Developing Skills

3. 30°, 150°, 270°
4. 30°, 150°, 210°, 330°
5. 60°, 120°, 240°, 300°
6. 45°, 135°, 225°, 315°
7. 60°, 300°
8. 90°, 210°, 270°, 330°
9. 45°, 63.4°, 225°, 243.4°
10. 0°, 70.5°, 289.5°
11. 19.5°, 41.8°, 138.2°, 160.5°
12. 66.4°, 113.6°, 246.4°, 293.6°
13. 63.4°, 99.5°, 243.4°, 279.5°
14. 70.5°, 75.5°, 284.5°, 289.5°
15. 1.11, 1.25, 4.25, 4.39
16. 0, 1.82, 4.46
17. 0, π (3.14), 3.55, 5.87
18. 3.48, 5.94
19. 0.25, 0.52, 2.62, 2.89
20. 0.17, 1.11, 3.31, 4.25
21. 30° or $\frac{\pi}{6}$
22. 2.03
23. −90°, 270°

13-3 Using the Quadratic Formula to Solve Trigonometric Equations *(page 534)*

Writing About Mathematics

1. When the discriminant is negative, the solutions are imaginary numbers.

2. When factored, $\csc \theta = 0$ and $\csc \theta = \frac{1}{2}$. The range of the cosecant function is $(-\infty, -1] \cup [1, \infty)$.

Developing Skills

3. 202°, 338°
4. 74°, 125°, 254°, 305°
5. 29°, 99°, 261°, 331°
6. 14°, 166°, 246°, 294°
7. 111°, 159°, 291°, 339°
8. 46°, 80°, 280°, 314°
9. 50°, 157°, 230°, 337°
10. 72°, 144°, 216°, 288°
11. 55°, 125°
12. 39°, 119°, 219°, 299°
13. 64°, 140°, 220°, 296°
14. { }
15. $\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$
16. 0.56, 5.72

13-4 Using Substitution to Solve Trigonometric Equations Involving More Than One Function *(pages 537–538)*

Writing About Mathematics

1. Yes. The maximum value of both $\sin\theta$ and $\cos\theta$ is 1, and they are never equal to 1 for the same value of θ. Therefore, their sum will always be less than 2.
2. $0° \le \theta \le 180°$. Sine is positive in the first and second quadrants.

Developing Skills

3. 30°, 150° **4.** 30°, 150°
5. 45°, 90°, 225°, 270° **6.** 30°, 150°, 270°
7. 0°, 60°, 300° **8.** 30°, 150°, 210°, 330°
9. 45°, 135°, 225°, 315° **10.** 90°, 270°
11. 45°, 135°, 225°, 315° **12.** 0°, 180°, 210°, 330°
13. 30°, 150° **14.** 45°, 135°, 225°, 315°

Applying Skills

15. a. (1) $2\sqrt{3} - 2$ in. (2) $2\sqrt{2} - 2$ in.
b. 0° **c.** ±28.96°

13-5 Using Substitution to Solve Trigonometric Equations Involving Different Angle Measures *(pages 540–541)*

Writing About Mathematics

1. No. Dividing by 2 divides the coefficients, not the angles.
2. No. You must account for the factor $\cos\theta$. The solution set also includes the numbers $\left\{\frac{\pi}{2}, \frac{3\pi}{2}\right\}$.

Developing Skills

3. 30°, 90°, 150°, 270° **4.** 0°, 180°, 360°
5. 0°, 180°, 360°
6. 30°, 90°, 150°, 210°, 270°, 330°
7. 30°, 150°, 210°, 330° **8.** 60°, 300°
9. 30°, 90°, 150° **10.** 45°, 135°, 225°, 315°
11. $\frac{\pi}{5}, \frac{3\pi}{5}, \frac{7\pi}{5}, \frac{9\pi}{5}$ **12.** $0, 1.82, \pi, 4.46, 2\pi$
13. $0.34, \frac{\pi}{2}, 2.80$ **14.** $0.17, \frac{\pi}{2}, 2.97, \frac{3\pi}{2}$
15. $0, 1.15, 1.99, \pi, 4.29, 5.13, 2\pi$
16. 1.36, 4.92 **17.** 0.28, 2.86 **18.** 0.12, 3.02

Applying Skills

19. a. $d = 90\sin\theta$
b. $d = 60\sin(90° - \theta)$
c. $90\sin\theta = 60\sin(90° - \theta)$

$$\sin\theta = \tfrac{2}{3}\cos\theta$$
$$\tan\theta = \tfrac{2}{3}$$
$$\theta = \arctan\tfrac{2}{3} = 33.69°$$

d. 49.92 m

20. a. $\sin\theta = \frac{y}{x}$, $\sin 2\theta = \frac{1.75y}{x} = \frac{7y}{4x}$
b.
$$\sin 2\theta = 1.75\sin\theta$$
$$2\sin\theta\cos\theta = 1.75\sin\theta$$
$$\sin\theta\,(2\cos\theta - 1.75) = 0,\ \text{reject}\ \sin\theta = 0$$
$$\cos\theta = \tfrac{1.75}{2} = \tfrac{7}{8}$$
$$\theta = \arccos\tfrac{7}{8} = 28.96°$$

Review Exercises *(page 543)*

1. 120°, 240° **2.** 240°, 300°
3. 60°, 300° **4.** 60°, 180°, 300°
5. 0°, 180, 360° **6.** 60°, 120°, 240°, 300°
7. 45°, 135°, 225°, 315° **8.** 45°, 135°, 225°, 315°
9. 22.5°, 202.5°
10. 30°, 90°, 150°, 210°, 270°, 330°
11. 3.43, 5.99 **12.** { }
13. $\frac{\pi}{2}, 3.39, 6.03$ **14.** 0, 2.30, 3.98
15. 1.34, 2.91, 4.48, 6.05 **16.** 1.20, 1.43, 4.85, 5.08
17. $0, 0.62, 2.53, \pi, 3.76, 5.67$
18. $\frac{\pi}{2}, 3.39, \frac{3\pi}{2}, 6.03$ **19.** 1.33, 4.47
20. $1.23, \frac{2\pi}{3}, \frac{4\pi}{3}, 5.05$ **21.** 1.11, 1.77, 4.25, 4.91
22. $\frac{\pi}{3}, \frac{5\pi}{3}$
23. The left side of the equation is equal to zero only at values of θ for which both the tangent and secant functions are undefined.

$$\tan\theta - \sec\theta = \frac{\sin\theta}{\cos\theta} - \frac{1}{\cos\theta} = 0$$
$$\sin\theta = 1$$
$$\theta = \tfrac{\pi}{2}$$

However, tangent is undefined at $\theta = \frac{\pi}{2}$.

24. a. $\tan\theta = \frac{CD}{AD}$
$\tan 2\theta = \frac{CD}{DB}$
b. $CD = \frac{\sqrt{5}}{5}AD$
c. $\tan\theta = \frac{CD}{AD} = \frac{\sqrt{5}}{5}$
d. $\theta = \arctan\frac{\sqrt{5}}{5} = 24.01°$
e. $m\angle A = 24°$, $m\angle B = 48°$, $m\angle C = 108°$

Exploration *(page 544)*

(1) **a.** $A = 4\tan\theta$ **b.** 0.12
(2) **a.** $A = 2\tan\theta$ **b.** 0.24
(3) **a.** $A = \frac{1}{2}\sin 2\theta = \sin\theta\cos\theta$ **b.** $\frac{\pi}{4}$
(4) **a.** $A = \frac{1}{2}\sin\theta$ **b.** $\frac{\pi}{2}$
(5) **a.** Area of triangle

$$= \tfrac{1}{2}bh$$
$$= \tfrac{1}{2}\sin(\pi - \theta)\cos(\pi - \theta) = -\tfrac{1}{2}\sin\theta\cos\theta$$

Area of semicircle $= \frac{\pi}{2}$

Total area $= \frac{\pi}{2} - \frac{1}{2}\sin\theta\cos\theta$

b. No possible value for θ. Since the area of the semicircle circle is $\frac{\pi}{2} \approx 1.57$ square units, the minimum area of the shaded region is $\frac{\pi}{2} > 1$ square units.

(6) **a.** $A = 2\sin\theta\cos\theta = \sin 2\theta$

b. $\frac{\pi}{12}$

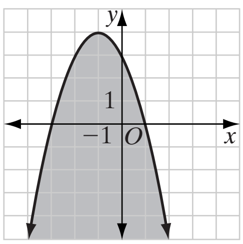
Cumulative Review *(pages 545–546)*

Part I

1. 3 **2.** 1 **3.** 4
4. 1 **5.** 2 **6.** 3
7. 4 **8.** 4 **9.** 2
10. 1

Part II

11.



12.
$$x = 4y - 2$$
$$x + 2 = 4y$$
$$\frac{x+2}{4} = y$$
$$f^{-1}(x) = \frac{x+2}{4}$$

Part III

13. $2, 2\sqrt[3]{9}, 2\sqrt[3]{81}, 18, 18\sqrt[3]{9}, 18\sqrt[3]{81}$

14. Center $= \left(\frac{-2+4}{2}, \frac{5-1}{2}\right) = (1, 2)$

Radius $= \sqrt{(-2-4)^2 + (5+1)^2} = \sqrt{36+36}$
$= 6\sqrt{2}$

$(x-1)^2 + (y-2)^2 = 72$

Part IV

15. $\sin\theta = \frac{1}{3}$ $\cos\theta = -\frac{2\sqrt{2}}{3}$

$\tan\theta = -\frac{\sqrt{2}}{4}$ $\cot\theta = -2\sqrt{2}$

$\sec\theta = -\frac{3\sqrt{2}}{4}$

16.
$$\frac{6}{x} - 2 = \frac{5}{x^2}$$
$$-2 = \frac{5}{x^2} - \frac{6}{x}$$
$$-2 = \frac{5-6x}{x^2}$$
$$0 = 2x^2 - 6x + 5 = 0$$
$$x = \frac{6 \pm \sqrt{36 - 4(2)(5)}}{4}$$
$$x = \frac{3}{2} \pm \frac{1}{2}i$$

Chapter 14. Trigonometric Applications

14-1 Similar Triangles *(pages 551–552)*

Writing About Mathematics

1. Yes, since $\tan\theta = \frac{-8.48}{-5.30} = 1.6$, θ can be found by using arctan.

2. Quadrant III. Both cosine and sine are negative when evaluated.

Developing Skills

3. $(2\sqrt{2}, 2\sqrt{2})$ **4.** $(\sqrt{3}, 1)$
5. $(0, 6)$ **6.** $(-4, 4\sqrt{3})$
7. $\left(-\frac{15}{2}\sqrt{2}, \frac{15}{2}\sqrt{2}\right)$ **8.** $(-0.5, 0)$
9. $\left(-\frac{9}{2}\sqrt{3}, \frac{9}{2}\right)$ **10.** $\left(-\frac{25}{2}\sqrt{3}, -\frac{25}{2}\right)$
11. $(0, -12)$ **12.** $(-1, -1)$
13. $\left(\frac{\sqrt{3}}{2}, -\frac{3}{2}\right)$ **14.** $(1, -\sqrt{3})$

15. a. 10 **b.** 53°
16. a. 13 **b.** 113°
17. a. 7 **b.** 90°
18. a. 15 **b.** 323°
19. a. 15 **b.** 0°
20. a. 14.42 **b.** 236°
21. a. 25 **b.** 16°
22. a. 11.66 **b.** 301°
23. a. 11.31 **b.** 135°
24. a. $R(5, 0), S\left(1.5, \frac{3\sqrt{3}}{2}\right)$ **b.** $\frac{15\sqrt{3}}{4}$ sq units
25. a. $R(12, 0), S(0, 8)$ **b.** 48 sq units
26. a. $R(8, 0), S\left(-4\sqrt{2}, 4\sqrt{2}\right)$ **b.** $16\sqrt{2}$ sq units
27. a. $R(20, 0), S(10, 10)$ **b.** 100 sq units
28. a. $R(9, 0), S(4.5, 4.5\sqrt{3})$ **b.** $20.25\sqrt{3}$ sq units
29. a. $R(7, 0), S(8\sqrt{3}, 8)$ **b.** 28 sq units

14-2 Law of Cosines *(pages 555–556)*

Writing About Mathematics

1. Let C be the obtuse angle of $\triangle ABC$. By the Law of Cosines, $c^2 = a^2 + b^2 - 2ab\cos C$. Since the cosine of an obtuse angle is negative, $-(2ab\cos C)$ is positive. Therefore, $c^2 = a^2 + b^2 + |2ab\cos C|$. A whole is greater than the sum of its parts, so $c^2 > a^2$ or $c > a$, and $c^2 > b^2$ or $c > b$.
2. The cosine of a right angle is zero. Thus, when c is the hypotenuse, $c^2 = a^2 + b^2 - 2ab\cos C = a^2 + b^2$, which is the Pythagorean theorem.

Developing Skills

3. $m^2 = a^2 + r^2 - 2ar\cos M$
4. $p^2 = n^2 + o^2 - 2no\cos P$

5. $2\sqrt{7}$ 6. $\sqrt{37}$ 7. $8\sqrt{2}$
8. 4 9. $\sqrt{26}$ 10. $\sqrt{13}$
11. $2\sqrt{19}$ 12. $9\sqrt{7}$ 13. $2\sqrt{10}$
14. 5.6 15. 147.0 16. 4.8
17. 98.6 18. 1.7 19. 7.5

Applying Skills

20. **a.** 0.72 mi **b.** 1.70 mi
21. 24.08 lb
22. **a.** 87 m **b.** 74 m
23. 28 ft 24. 151.1 m
25. 36.5 nautical miles
26. $$\begin{aligned} c^2 &= 2x^2 - 2x^2\cos 60^\circ \\ &= 2x^2 - x^2 \\ &= x^2 \\ \therefore c &= x \end{aligned}$$

14-3 Using the Law of Cosines to Find Angle Measure *(pages 558–559)*

Writing About Mathematics

1. Let $\angle C$ be the angle opposite the side of length 12. Then:
$$\begin{aligned} 12^2 &= 4^2 + 7^2 - 2(4)(7)\cos\angle C \\ 79 &= -56\cos\angle C \\ \cos\angle C &= -\tfrac{79}{56} < -1 \end{aligned}$$
No $\angle C$ exists such that $\cos\angle C$ is less than -1.
2. $$\begin{aligned} c^2 &= a^2 + b^2 - 2ab\cos C \\ c^2 + 2ab\cos C &= a^2 + b^2 \end{aligned}$$
If $\angle C$ is obtuse, $2ab\cos C$ is negative; thus $c^2 > a^2 + b^2$.

Developing Skills

3. $\cos T = \frac{u^2 + v^2 - t^2}{2uv}$ 4. $\cos Q = \frac{p^2 + r^2 - q^2}{2pr}$
5. -0.575 6. 0
7. $\cos A = \frac{7}{8}$, $\cos B = \frac{11}{16}$, $\cos C = -\frac{1}{4}$
8. $\cos A = -\frac{1}{8}$, $\cos B = \frac{3}{4}$, $\cos C = \frac{3}{4}$
9. $\cos D = -\frac{17}{192}$, $\cos E = \frac{29}{48}$, $\cos F = \frac{61}{72}$
10. $\cos P = \frac{37}{40}$, $\cos Q = \frac{13}{20}$, $\cos R = -\frac{5}{16}$
11. $\cos M = \frac{11}{80}$, $\cos N = \frac{95}{256}$, $\cos P = \frac{139}{160}$
12. $\cos A = \frac{12}{13}$, $\cos B = \frac{5}{13}$, $\cos C = 0$
13. 33°, 64°, 83° 14. 36°, 40°, 104°
15. 42°, 51°, 87° 16. 47°, 47°, 86°
17. 48°, 63°, 69° 18. 16°, 74°, 90°
19. 37° 20. 122°
21. **a.** 33.7 in. **b.** 58°
22. 83° 23. 82°
24. Let x = the length of any side of the equilateral triangle.
Let $\angle C$ = any angle of the triangle.
$$\begin{aligned} \cos\angle C &= \frac{x^2 + x^2 - x^2}{2x^2} \\ \cos\angle C &= \frac{x^2}{2x^2} \\ \cos\angle C &= \frac{1}{2} \\ \angle C &= 60^\circ \end{aligned}$$

14-4 Area of a Triangle *(pages 563–564)*

Writing About Mathematics

1. Since $\angle A$ and $\angle B$ are supplementary, $\sin A = \sin(180^\circ - A) = \sin B$.
2. Yes. The area of the rhombus is $(PQ)(PS)(\sin P)$. Since the sides are congruent, $(PQ)(PS)(\sin P) = (PQ)^2(\sin P)$.

Developing Skills

3. 3 sq units 4. 30 sq units
5. 60 sq units 6. 108 sq units
7. 16.8 sq units 8. 12 sq units
9. 77.5 sq units 10. 24,338.5 sq units
11. 12.6 sq units 12. 25,221.0 sq units
13. 122.0 sq units 14. 36,615.3 sq units
15. $400\sqrt{3}$ m² 16. $36\sqrt{2}$ cm²
17. 480 ft²

Applying Skills

18. **a.** $\frac{1}{3}$ **b.** $\frac{2\sqrt{2}}{3}$
c. $4\sqrt{2}$ km² **d.** 6 km²
19. 234 ft² 20. $125\sqrt{3}$ ft²

21. $\sin 30° = \sin(180° - 30°) = \sin 150°$
Thus, $\frac{1}{2}(AB)(BC)\sin 30° = \frac{1}{2}(DE)(EF)\sin 150°$.

22. a. $\frac{2}{3}$ **b.** 41.8° or 138.2°
c. Yes, an acute or an obtuse triangle.

23. a. $A = ac\sin\theta$ **b.** $\theta = 90°$

14-5 Law of Sines *(pages 567–568)*

Writing About Mathematics

1. No. A positive sine means the angle is in the first or third quadrant; that is, the angle can be either acute or obtuse.

2. Yes. A positive cosine means the angle is in the first or fourth quadrant. However, in a triangle, each angle when drawn in standard position is in the first or second quadrant. Thus, when the cosine is positive, the angle is acute. Similarly, a negative cosine means the angle is obtuse.

Developing Skills

3. $3\sqrt{6}$ **4.** 48 **5.** $4\sqrt{3}$
6. $4\sqrt{6}$ **7.** 12.5 **8.** $\frac{64}{3}$
9. 23.5 **10.** 31.4 **11.** 44.5
12. 18.3 **13.** 97.7 **14.** 16.9
15. 6.93
16. a. 8.85 cm **b.** 32.2 cm
17. a. 31.1 in. **b.** 83 in.
18. $\frac{a}{\sin A} = \frac{c}{\sin 90°}$
$\frac{a}{\sin A} = \frac{c}{1}$
$\sin A = \frac{a}{c}$

Applying Skills

19. a. 3.18 ft **b.** 12.3 ft
20. 138.0 ft, 250.2 ft
21. a. 14.0 ft **b.** 18.6 ft
22. 3.1 mi **23.** \$5,909

14-6 The Ambiguous Case *(pages 573–574)*

Writing About Mathematics

1. Yes, the side opposite the largest angle, 110°, has to be the largest side of the triangle. Since it is not, no triangle can exist with these measurements.

2. Because we know that one of the angles is obtuse, we cannot use the ambiguity of the sine function to imply two possible triangles.

Developing Skills

3. a. 2 **b.** {20°, 155°, 5°}, {20°, 25°, 135°}
4. a. 1 **b.** {30°, 60°, 90°}
5. a. 1 **b.** {39°, 49°, 92°}
6. a. 0
7. a. 1 **b.** {29°, 31°, 120°}
8. a. 1 **b.** {3°, 27°, 150°}
9. a. 0
10. a. 2 **b.** {15°, 20°, 145°}, {5°, 15°, 160°}
11. a. 0
12. a. 1 **b.** {135°, 30°, 15°}
13. a. 1 **b.** {30°, 60°, 90°}
14. a. 2 **b.** {45°, 62°, 73°}, {45°, 17°, 118°}

Applying Skills

15. a. 60.07°
b. No, the triangle formed by the ladder, wall, and ground is a right triangle.

16. Yes, there can be only one garden.
Angles: {37°, 68°, 75°}
Sides: {5 ft, 7.7 ft, 8 ft}

17. No. Since $10 < 12$, 10 cm must be the length of the short diagonal. Therefore, the other angle measures 60°. Using the Law of Sines to find the angle opposite the 12 cm side yields a value of sine greater than 1.

18. Yes. Two triangles are possible.
Sides of 1st triangle: {2.0 km, 2.5 km, 2.7 km}
Sides of 2nd triangle: {2.0 km, 2.5 km, 0.8 km}
The route corresponding to the first triangle is longer.

14-7 Solving Triangles *(pages 579–580)*

Writing About Mathematics

1. Since $\triangle BCD$ is a right triangle:

$$BC = \frac{85}{\sin 35°} \approx 148.193 \text{ ft}$$

Use the Law of Sines in $\triangle BCA$:

$$\frac{BA}{\sin 40°} = \frac{148.193}{\sin 105°}$$
$$BA \approx 98.62 \text{ ft}$$

2. An angle of depression is the complement of the complement of the angle of elevation. Taking the complement of a complement is congruent to the original angle.

Developing Skills

3. a. Law of Cosines **b.** 4.9
4. a. Law of Sines **b.** 59.0°, 121.0°
5. a. Law of Cosines **b.** 11.6°
6. a. Law of Cosines **b.** 76.9°
7. a. Both **b.** 115.7°
8. a. Law of Cosines **b.** 122.6°
9. a. Law of Sines **b.** 12.7°
10. a. Law of Sines **b.** 8.2
11. $c = 17, \angle A = 51°, \angle B = 69°$
12. $c = 20, \angle A = 25°, \angle C = 125°$
13. $a = 34, c = 31, \angle A = 75°$
14. $b = 5, c = 7, \angle A = 90°$
15. $f = 99, \angle D = 43°, \angle E = 27°$

16. $q = 13, r = 6, \angle P = 70°$
17. $\angle R = 109°, \angle S = 44°, \angle T = 27°$
18. $C = 36, \angle A = 28°, \angle B = 22°$
19. $\angle P = 37°, \angle Q = 53°, \angle R = 90°$
20. $f = 62, \angle E = 90°, \angle F = 60°$
21. $t = 23, \angle R = 40°, \angle S = 50°$
22. $\angle A = 25°, \angle B = 135°, \angle C = 20°$
23. 27.4

Applying Skills

24. a. 78.6°, 101.4°
b. 37.8°, 40.8°, 60.6°
c. 2.3 km

25. 107 ft **26.** 16.6 ft, 19.2 ft
27. 35 ft **28.** 55.6°, 71.4°, 116.5°, 116.5°

Review Exercises *(pages 582–584)*

1. 21 **2.** 17.5 in.
3. 50° **4.** 112°
5. $\cos\theta = \frac{10^2 + 24^2 - 26^2}{2(10)(24)} = 0$
$\therefore \theta = 90°$
6. a. 240 sq units **b.** 20
7. a. 31.3° **b.** 42.8 sq units
8. $AB = AC = 26.2$ **9.** 15, 21
10. a. 2
b. $\angle B = 58°, \angle C = 74°$
c. $\angle B = 122°, \angle C = 10°$
11. $\sin R = \frac{r \sin P}{p} = \frac{15 \sin 66°}{12} = 1.14 > 1$
12. a. 53°, 127° **b.** 25.0 in.
c. Using the answers to part **a**: 626 in.^{2}
13. $49 = 64 + c^2 - (16)(c)\cos 60°$
$0 = c^2 - 8c + 15$
$c = 3, 5$
14. $\sin B = \frac{b \sin A}{a} = \frac{8 \sin 60°}{7} = \frac{4\sqrt{3}}{7}$
$m\angle B = 81.8°$
$m\angle B = 180° - 81.8° = 98.2°$
15. 25.4 ft
16. From A: 8.7 mi, from B: 7.0 mi
17. a. $\frac{24}{25}$ **b.** 37°
c. Using the answer to part **b**: 13.9

Exploration *(pages 584–585)*

Part A

Answers will vary. $\triangle DEF$ is an equilateral triangle.

Part B

Steps 1–8. Answers will vary.
Step 9. Yes, $\triangle DEF$ is an equilateral triangle.

Cumulative Review *(pages 585–586)*

Part I

1. 2	**2.** 1	**3.** 4
4. 3	**5.** 1	**6.** 2
7. 3	**8.** 3	**9.** 3
10. 3		

Part II

11.
$$\begin{aligned}\cos 15° &= \cos(45° - 30°)\\ &= \cos 45° \cos 30° + \sin 45° \sin 30°\\ &= \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2}\cdot\frac{1}{2}\\ &= \frac{\sqrt{6} + \sqrt{2}}{4}\end{aligned}$$

12.
$$\begin{aligned}&\frac{1 - \sqrt{5}}{1 + \sqrt{5}}\cdot\frac{1 - \sqrt{5}}{1 - \sqrt{5}}\\ &= \frac{1 - 2\sqrt{5} + 5}{1 - 5}\\ &= \frac{6 - 2\sqrt{5}}{-4}\\ &= \frac{\sqrt{5} - 3}{2}\end{aligned}$$

Part III

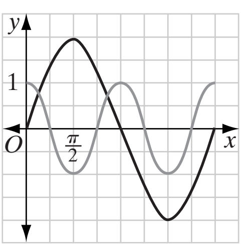
13. *Answer:* $x = 2$
$$\begin{aligned}3 - \sqrt{2x - 3} &= x\\ 3 - x &= \sqrt{2x - 3}\\ x^2 - 6x + 9 &= 2x - 3\\ x^2 - 8x + 12 &= 0\\ (x - 6)(x - 2) &= 0\end{aligned}$$
$x = 6$ | $x = 2$
Reject extraneous root

14. *Answer:* {210°, 330°}
$$\begin{aligned}6\sin^2 x - 5\sin x - 4 &= 0\\ \left(x + \tfrac{1}{2}\right)\left(x - \tfrac{4}{3}\right) &= 0\end{aligned}$$
$\sin x = -\frac{1}{2}$ | $\sin x = \frac{4}{3}$
$x = 210°, 330°$ | Reject extraneous root

Part IV

15.
$$\begin{aligned}\log_b x &= \log_b\left(\frac{3 \cdot 4^2}{\sqrt{8}}\right)\\ x &= \frac{3(16)}{2\sqrt{2}}\\ x &= 12\sqrt{2}\end{aligned}$$

16. a–b.



c. 2

Chapter 15. Statistics

15-1 Gathering Data *(pages 594–595)*

Writing About Mathematics

1. A control group is necessary to ensure that any changes to members of the experimental group are due to the medication and not to some external factor, such as the placebo effect.
2. It is necessary that a participant does not know to which group he or she belongs because this knowledge can influence the participant's perception of the effectiveness of the treatment.

Developing Skills

3.

Stem	Leaf
9	0 0 2 5
8	2 4 5 5 6 7 8
7	4 5 6 8
6	6 7 7 8
5	4

Key: 5 | 4 = 54

4.

Stem	Leaf
23	6
22	4
21	0
20	1 3 5 7
19	0 2 5 5 6 6
18	2 4 7 8
17	3 5 7
16	9
15	5

Key: 15 | 5 = 155

5.

Stem	Leaf
15	1
14	0 1 2 2 3 4 4 7
13	0 1 1 2 2 4 6 7 7 7 7 8 9
12	7 9 9

Key: 12 | 9 = 129

6.

No. of Books Read	Frequency
8–9	1
6–7	3
4–5	5
2–3	9
0–1	7

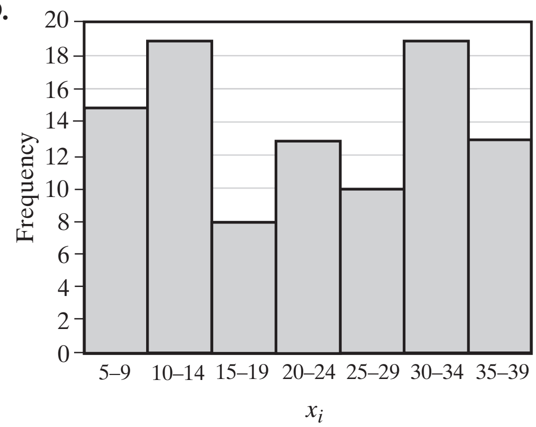
7.

Size	Frequency
15–17	3
12–14	11
9–11	7
6–8	5

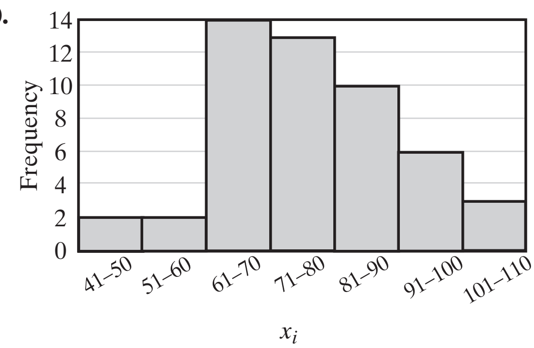
8.

No. of Siblings	Frequency
6–7	1
4–5	2
2–3	11
0–1	16

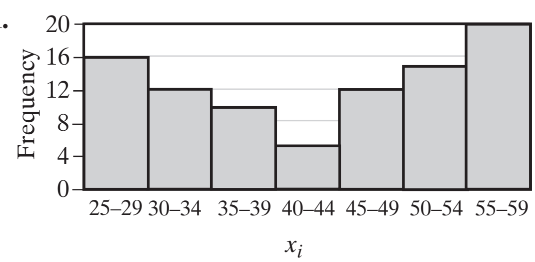
9.



10.



11.



Applying Skills

In 12–18, answers will vary.

12. Record a sample of the temperature twice daily, at perceived high and low temperatures, and take the average over each month.

13. Conduct a survey on a sample, such as every tenth person leaving the restaurant on a given day.

14. Conduct an observational study on a sample, such as recording the patient's temperature every 2 hours.

15. Conduct a census on the population, recording all students' grades on the test.

16. Conduct a census on the population, counting the number of people living in each house, apartment, etc. (Note that because of the size of the population, methods involving random samples will need to be used.)

17. Conduct a survey on a sample, such as measuring the height of every fifth student enrolling in kindergarten in each elementary school.

18. Conduct a survey on a sample population of moviegoers, such as questioning every tenth person leaving 100 randomly selected theaters where the movie is showing.

19. a.

Stem	Leaf
9	2 3 5 8 8
8	2 3 4 4 6 6 7 7 8 9
7	2 4 5 6 7 7
6	1 6
5	3 8

Key: 5 | 3 = 53

b.

Score	Frequency
91–100	5
81–90	10
71–80	6
61–70	2
51–60	2

c. 21 **d.** 2

20. a. 2 **b.** 3 **c.** 12
d. 72 **e.** 10

Hands-On Activity

Answers will vary.

15-2 Measures of Central Tendency
(pages 604–605)

Writing About Mathematics

1. No. Whenever the number of data values of a set is odd, the number of data values less than the lower quartile or greater than the upper quartile cannot total exactly 50% of the number of data values.

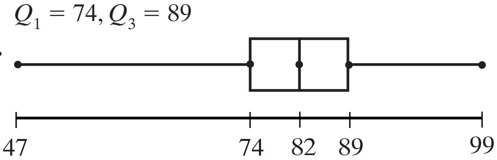
2. Yes. Whether a set has $2n$ or $2n + 1$ data values, there are n data values above the median and n data values below the median.

Developing Skills

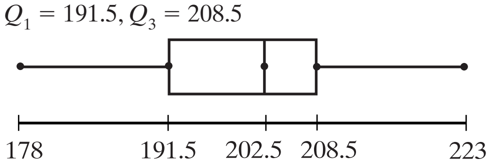
3. Mean = $81.\overline{1}$, median = 80, mode = 80
4. Mean = 66, median = 65.5, mode = 68
5. Mean = 122.4, median = 117, modes = 115, 118
6. Mean = 2.4, median = 2, modes = 0, 2
7. Mean = \$8.26, median = \$7.88, mode = \$7.50
8. Mean = \$3.48, median = \$3.50, mode = \$5.00
9. $Q_1 = 6.5$, median = 15, $Q_3 = 21$
10. $Q_1 = 36$, median = 42.5, $Q_3 = 44$
11. $Q_1 = 19$, median = 26, $Q_3 = 28.5$
12. $Q_1 = 81$, median = 87, $Q_3 = 90.5$
13. $Q_1 = 58$, median = 62, $Q_3 = 66$
14. $Q_1 = 19.5$, median = 26, $Q_3 = 30$
15. a. 90 **b.** 92 or 95
c. Any number other than 90, 92, or 95
16. $Q_1 = 25.5$, $Q_2 = 50.5$, $Q_3 = 75.5$
17. $Q_1 = 24.5$, $Q_2 = 50$, $Q_3 = 75.5$

Applying Skills

18. a. 79.52 **b.** 82
c. $Q_1 = 74$, $Q_3 = 89$
d.



19. a. 200.7 **b.** 202.5 **c.** 202
d. $Q_1 = 191.5$, $Q_3 = 208.5$
e.



20. 83

Hands-On Activity

Answers will vary.

15-3 Measures of Central Tendency for Grouped Data *(pages 611–614)*

Writing About Mathematics

1. Not necessarily. If the ages are not distributed perfectly evenly, then Adelaide cannot make this assumption.

2. Yes. There are only six possible ages those employees could be, so there must be some employees with the same age.

Developing Skills

3. Mean = $3.\overline{08}$, median = 3, mode = 3
4. Mean = 32.4, median = 30, mode = 30
5. Mean = 8.84, median = 9, mode = 9
6. Mean = $6.\overline{63}$, median = 7, mode = 7
7. Mean = $1.34, median = $1.30, mode = $1.30
8. Mean = $81.\overline{6}$, median = 80, mode = 85
9. 19th percentile
10. 16th percentile
11. 36th percentile
12. 32nd percentile
13. Mean = 12.8, median = 12.6
14. Mean = 78.5, median = 78.5
15. Mean = $1.3, median = $1.3
16. Mean = 11.3, median = 10.9
17. Mean = $33.9, median = $27.50
18. Mean = 0.2, median = 0.2

Applying Skills

19. Mean = 11.625, median = 12, mode = 12
20. Mean = 17.43, median = 17
21. **a.** Mean = 35.45, median = 35
b. 25th percentile
22. Mean = 251.875, median = 253.58
23. Mean = 48.8, median = 51.125

Hands-On Activity

Answers will vary.

15-4 Measures of Dispersion *(pages 617–619)*

Writing About Mathematics

1. No. The subscript for each data value indicates its position in a list of data values, not its value.

2. Yes. An outlier is a data value that is 1.5 times the interquartile range below the first quartile or above the third quartile. For the given information, the interquartile range is 6, and $12 - (1.5)(6) = 3$, which makes the data value 2 an outlier.

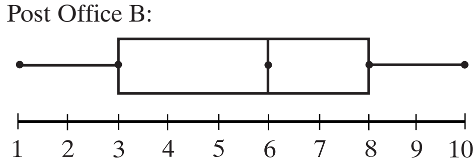
Developing Skills

3. Range = 16
Interquartile range = 10
4. Range = 22
Interquartile range = 8
5. Range = 81
Interquartile range = 53
6. Range = 40
Interquartile range = 4
7. Mean = 35, median = 35, range = 30, interquartile range = 10
8. Mean = 7, median = 7, range = 8, interquartile range = 2
9. Mean = 24.9, median = 25, range = 26, interquartile range = 17.5

Applying Skills

10. **a.** Mean = 2,296.5, median = 480
b. The median is more representative. The mean is strongly influenced by the outlier.
c. The outlier is 19,014.
d. With outlier removed, mean = 439 and median = 427.
e. The mean is more representative of the data with the outlier removed.
11. **a.** 75.6 **b.** 80
c. $Q_1 = 65$, $Q_3 = 88.5$
d. 53 **e.** 23.5
12. Range = 4, interquartile range = 2
13. Range = 9, interquartile range = 2.5
14. **a.** 14 **b.** 3 **c.** 15
15. Range = 1, interquartile range = 0.4
16. **a.** Range of $A = 9$ = range of $B = 9$; yes, they are the same.
b. Post Office A:

Post Office B:



c. Interquartile range of $A = 1$, interquartile range of $B = 5$
d. Post Office A. Wait times there of 9 and 10 minutes are outliers, which is not the case at Post Office B.

Hands-On Activity

Answers will vary.

15-5 Variance and Standard Deviation *(pages 625–627)*

Writing About Mathematics

1. The second data set (from the sample) has the larger standard deviation since its denominator is smaller.
2. Yes. If the standard deviation is the square root of the variance, then the variance is the square of the standard deviation.

Developing Skills

3. Variance ≈ 3.92
   Standard deviation ≈ 1.98
4. Variance ≈ 8.29
   Standard deviation ≈ 2.88
5. Variance ≈ 116.74
   Standard deviation ≈ 10.80
6. Variance ≈ 1,223.14
   Standard deviation ≈ 34.97
7. Variance ≈ 66.24
   Standard deviation ≈ 8.14
8. Variance ≈ 6.65
   Standard deviation ≈ 2.58
9. Variance ≈ 233.36
   Standard deviation ≈ 15.28
10. Variance ≈ 32.99
    Standard deviation ≈ 5.74
11. Variance ≈ 877.38
    Standard deviation ≈ 29.62
12. Variance ≈ 12.57
    Standard deviation ≈ 3.55
13. Variance ≈ 648.99
    Standard deviation ≈ 25.48
14. Variance ≈ 106.78
    Standard deviation ≈ 10.33
15. Variance ≈ 3.23
    Standard deviation ≈ 1.80
16. Variance ≈ 4.20
    Standard deviation ≈ 2.05

Applying Skills

17. Line A. Since its standard deviation is smaller, its late times are more closely clustered around the mean of 10 minutes.
18. **a.** Variance ≈ 269.43
    b. Standard deviation ≈ 16.41
19. Variance ≈ 0.66; standard deviation ≈ 0.81

20. 2.21 **21.** 2.14 **22.** 5.93

Hands-On Activity

Answers will vary.

15-6 Normal Distribution *(pages 632–634)*

Writing About Mathematics

1. The mean of these scores is 90. In a normal distribution, 50% of the scores are below the mean. Only one of these five scores is below 90.
2. No. In a normal distribution the intervals closest to the mean contain more of the scores. Scores are not uniformly distributed through the first standard deviation above the mean.

Developing Skills

3. 68% **4.** 81.5% **5.** 81.5%
6. 84% **7.** 84% **8.** 50%
9. 50%
10. a. 45 **b.** 52 **c.** 35 **d.** 28

Applying Skills

11. (4) **12.** (2) **13.** (3) **14.** (3)

15. **a.** About 0.62%. Use normalcdf(0, 16, 16.1, 0.04).
    b. About 98.8%.
    Use normalcdf (16, 16.2, 16.1, 0.04).
16. About 1.3% of the time. Ken can expect to be punctual approximately 98.7% of the time. This means he will be late approximately 100% − 98.7% = 1.3% of the time.
17. About 8.96% of patrons check out more than 7 books. Approximately 91.04% of patrons check out fewer than 7 books, so 100% − 91.04% = 8.96% of patrons check out more than 7 books.

18. −0.5 **19.** 8

20. The science test. On the math test, Nora's score was within 2 standard deviations of the mean. On the science test, her score was more than 3 standard deviations above the mean.

15-7 Bivariate Statistics *(pages 638–640)*

Writing About Mathematics

1. Univariate data consists of one number for each data point, or a single set of numbers. Bivariate data consists of two numbers for each data point, or two different sets of numbers. Example answers will vary.
2. A positive slope reflects a positive correlation and a negative slope reflects a negative correlation. Slope cannot be used to measure the strength of a correlation.

Developing Skills

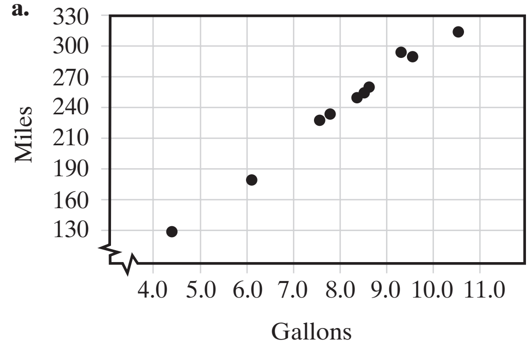
3. Bivariate **4.** Univariate
5. Bivariate **6.** Univariate
7. Moderate linear correlation
8. No linear correlation

9. Strong linear correlation

10. No linear correlation

Applying Skills

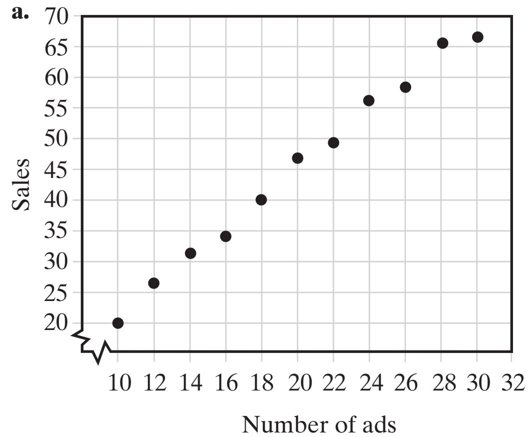
11. a.



b. Strong positive linear correlation

c. $y = 30.714x - 4.166$

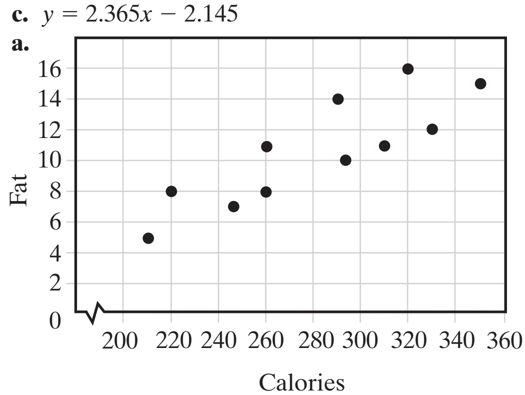
12. a.



b. Strong positive linear correlation

c. $y = 2.365x - 2.145$

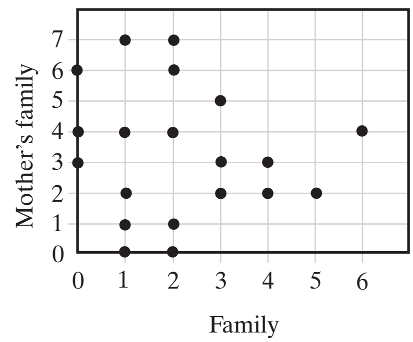
13. a.



b. Moderate positive linear correlation

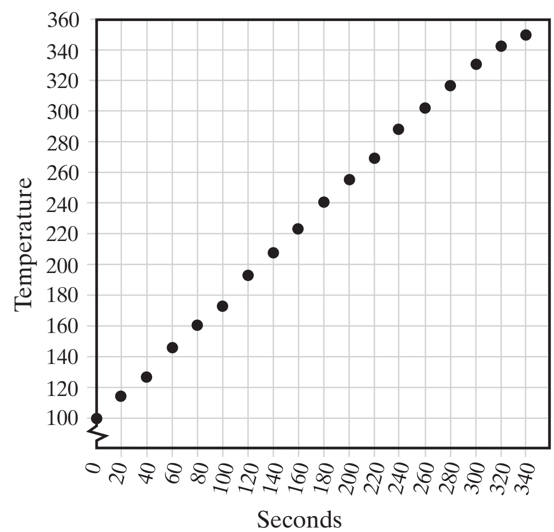
c. $y = 0.065x - 7.681$

14. a.



b. No linear correlation

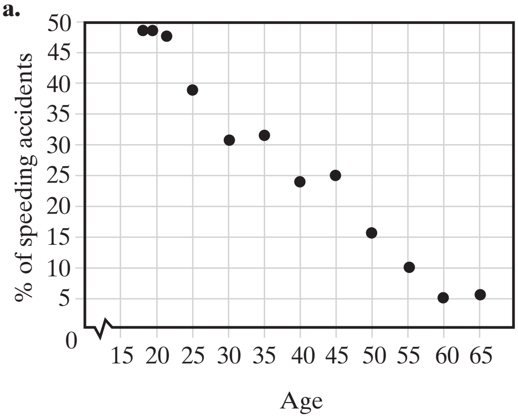
15. a.



b. Strong positive linear correlation

c. $y = 0.765x + 99.480$

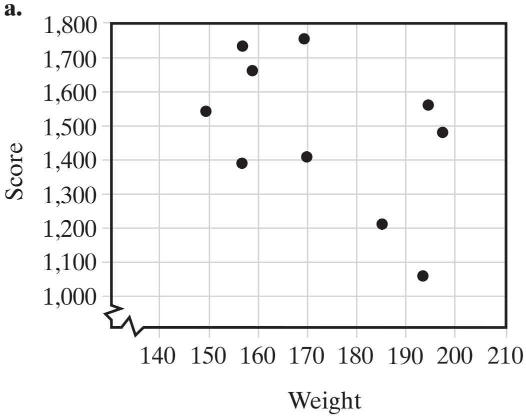
16. a.



b. Strong negative linear correlation

c. $y = -0.965x + 64.990$

17. a.



b. Moderate negative linear correlation

c. $y = -6.043x + 2{,}527.114$

15-8 Correlation Coefficient

(pages 645–646)

Writing About Mathematics

1. No. When $|r| = 1$ there is a perfect linear relationship between the data values, while a correlation coefficient of 0 indicates no linear relationship exists between the data values.

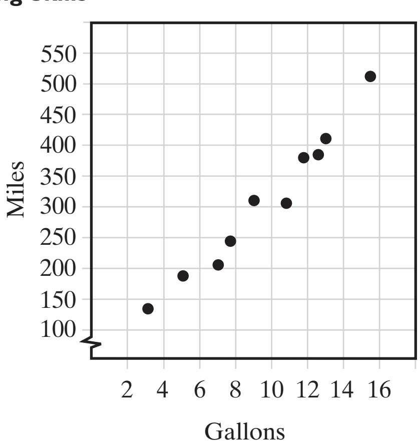
2. 1. There is a perfect linear relationship between temperature measured in degrees Fahrenheit and measured in degrees Celsius (otherwise they wouldn't be measuring the same thing!).

Developing Skills

3. 1	**4.** 0
5. −1	**6.** 0
7. Strong positive	**8.** Strong negative
9. None	**10.** Moderate/weak positive
11. Strong positive	**12.** Moderate negative
13. None	**14.** Strong negative

Applying Skills

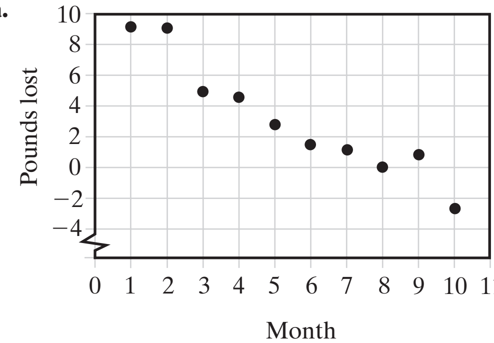
15. a.



b. Close to 1. There appears to be a very strong positive linear correlation.

c. $r = 0.99$

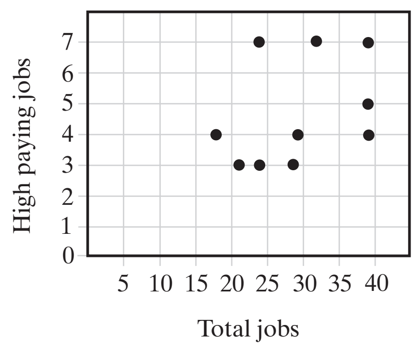
16. a.



b. Close to −1. There appears to be a very strong negative linear correlation.

c. $r = -0.96$

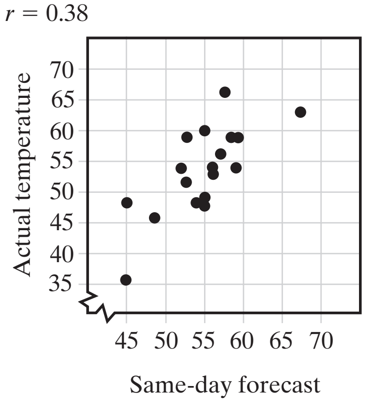
17. a.



b. Close to 0. There does not appear to be a strong correlation.

c. $r = 0.38$

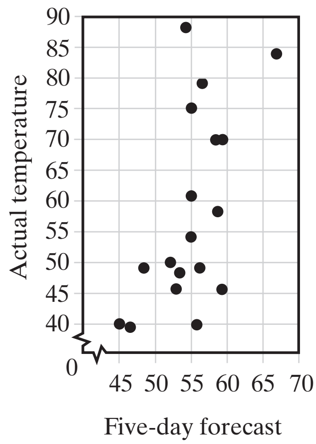
18. a.



b. Closer to 1. There appears to be a moderate positive linear correlation.

c. $r = 0.75$

19. a.



b. Closer to 1. There appears to be a moderate positive linear correlation.

c. $r = 0.59$

20. a. 1. There would be a perfect positive linear correlation.

b. Greater than. Yes. Same-day forecasts should be more accurate than forecasts for five days in the future.

15-9 Non-Linear Regression
(pages 651–654)

Writing About Mathematics

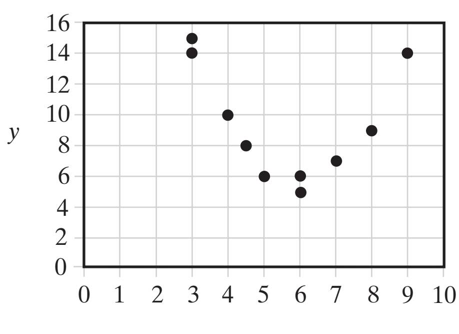
1. The function $y = \ln x$ is undefined for $x = 0$.

2. Function $y = ax^b$ has only positive or only negative y-values when b is even. If b is odd, the power function will have both positive and negative y-values.

Developing Skills

3. Quadratic
4. Exponential
5. Logarithmic
6. Exponential
7. Power
8. Cubic

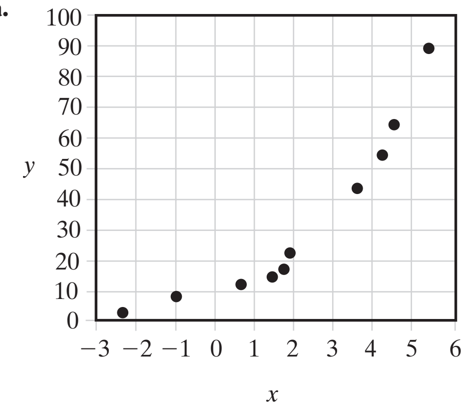
9. a.



b. Quadratic. The scatter plot appears to be quadratic.

c. $y = 0.968x^2 - 11.705x + 40.950$

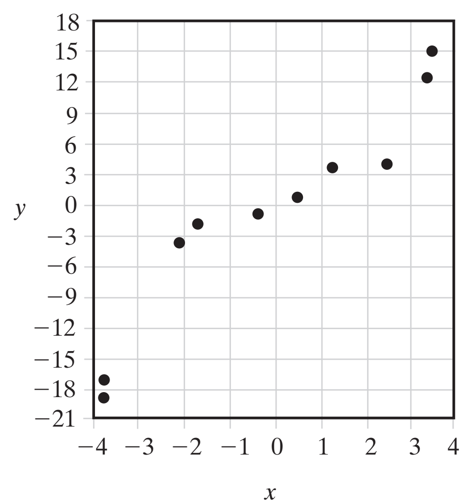
10. a.



b. Exponential. The scatter plot resembles an exponential curve. The curve does not pass through the origin, $y_i > 0$, and the y-intercept is positive.

c. $y = 8.609(1.560)^x$

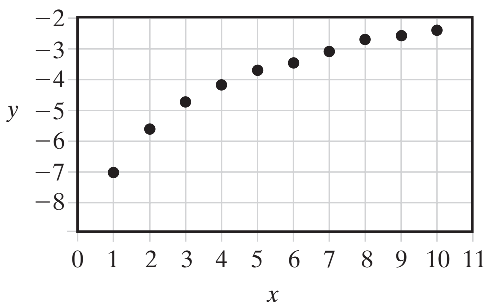
11. a.



b. Cubic. The scatter plot resembles a cubic curve.

c. $y = 0.291x^3 - 0.027x^2 + 0.557x + 0.467$

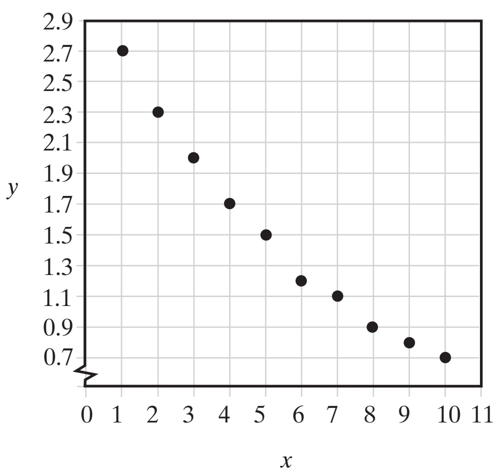
12. a.



b. Logarithmic. The scatter plot resembles a logarithmic curve that does not pass through the origin, $x_i > 0$, and the y-intercept appears negative.

c. $y = -6.995 + 2.003 \ln x$

13. a.



b. Exponential. The scatter plot resembles an exponential curve that does not pass through the origin, $y_i > 0$, and the y-intercept is positive.

c. $y = 3.127(0.859)^x$

Applying Skills

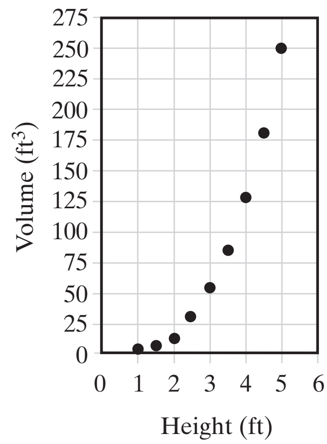
14. a. $y = 999.843(1.045)^x$ **b.** $1,623.00

15. a. $y = 19.165 + 5.026 \ln x$ **b.** 37.4 in.

16. a. The power regression equation, $y = 123{,}113.744x^{-1.981}$.

b. Yes. When Neptune's orbital speed is plugged into the regression equation, we get 4,277.2 million km as its distance from the sun, which is a reasonably good estimate.

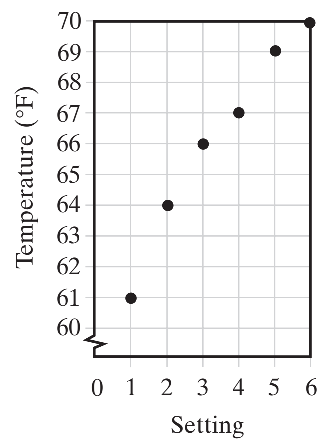
17. a.



b. Power regression. It resembles the positive half of a power function passing through (0, 0), $x_i > 0$, and $y_i > 0$.

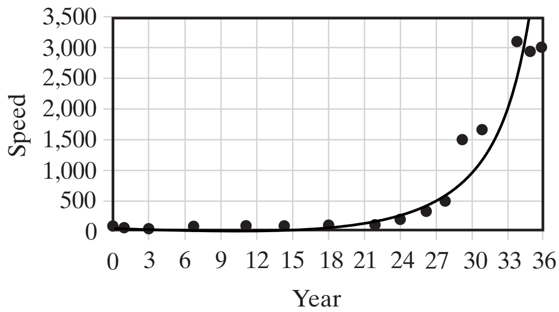
c. $y = 2.024x^{2.991}$

18. a.



b. $y = 60.811x^{0.076}$

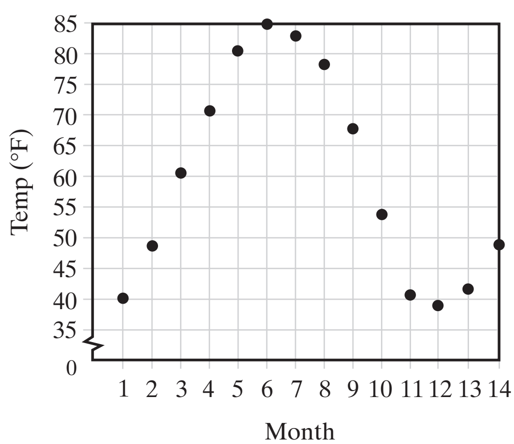
19.



The exponential regression model is
$y = 0.407(1.294)^x$.
No, Moore's Law does not appear to hold for Intel chips. According to this model, the speed of Intel computer chips increases by a factor of $1.294^2 \approx 1.67$ every two years.

Hands-On Activity: Sine Regression

a.



b. $y = 23.442 \sin (0.527x - 1.713) + 62.481$
c. 60°
d. 72°

15-10 Interpolation and Extrapolation *(pages 658–661)*

Writing About Mathematics

1. Interpolation is estimating a function value between given values. Extrapolation is estimating a function value outside the range of given values.
2. A major source of error when using extrapolation is that the regression model does not always hold outside of the given range of values.

Developing Skills

In 3–9, parts **b** and **c**, answers shown are obtained using the rounded regression equation from part **a**.

3. a. $y = 0.064x + 0.971$
b. 1.336 **c.** 4.359
4. a. $y = 0.486x + 2.607$
b. 8.439 **c.** 282.702
5. a. $y = 1.665x - 1.147$
b. -0.315 **c.** 0.989
6. a. Logarithmic: $y = 0.800 + 0.413 \ln x$
b. 0.939 **c.** 5.446
7. a. Quadratic: $y = 0.989x^2 - 39.930x + 407.290$
b. 70.546 **c.** 11.436, 28.938
8. a. Exponential: $y = 5.008(2.240)^x$
b. 23,837.928 **c.** 3.713
9. a. Power: $y = 0.578x^{2.716}$
b. 493.156 **c.** 8.840

Applying Skills

In 10–16, answers shown are obtained using the rounded regression equation.

10. The linear regression model is $y = 0.206x - 400.971$.
a. 10.63% **b.** 14.12%
11. The linear regression model is $y = 0.041x - 1.851$.
a. 5.53 sec **b.** 11.47 sec
12. The linear regression model is $y = 1.8x + 32$.
a. 77°F **b.** −20°C
13. Let x = the number of years since 1980. The exponential model is $y = 78.753(1.187)^x$.
a. 2,428 cars **b.** 56 cars
14. The power model is $y = 60.811x^{0.076}$.
a. 65.2°F **b.** 4.35°F
15. The power model is $y = 2.000x^{3.000}$ or $y = 2x^3$.
a. 3.91 ft^3 **b.** 3.7 ft **c.** 7.4 ft
16. a. The logarithmic model is $y = 6.784 + 5.063 \ln x$.
b. 10.3 in. **c.** 22.0 in.

Review Exercises *(pages 664–668)*

1. Univariate **2.** Bivariate **3.** Bivariate
4. Census: counting data of general interest for an entire population.
Survey: asking questions (oral or written) to find out experiences, preferences, or opinions.
Controlled experiment: structured study, usually of two groups, to compare results of a treatment or other process that only one group undergoes.
Observational study: structured study, usually of two groups, in which researchers do not impose the treatment on either group.

5. a. A sample. The student did not obtain information for every 9th grade student in the state.

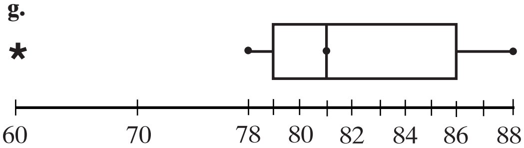
b. No. The data collected cannot be expected to reflect the grades of all students taking the test. The sample was very small and was not representative of the population as a whole, since the data was gathered from only one high school in the state.

6. a. 28 **b.** 80 **c.** 81

d. $Q_1 = 79, Q_3 = 86$ **e.** 7

f. Yes. The grade of 60 is an outlier since it is less than 1.5 times the interquartile range below the lower quartile: $79 - 1.5(7) = 68.5$.

g.



7. a. 38.2 **b.** 38 **c.** 38

d. 6 **e.** $Q_1 = 37, Q_3 = 39$

f. 2 **g.** 2.43 **h.** 1.56

i. The STAT menu on the calculator yields the same values as those found in parts **a**–**h**.

8. The sample mean is 84.49 seconds.
The sample variance is 1.957 seconds.

a. 70% **b.** 96%

c. The data appears to approximate a normal distribution. The data appears bell-shaped, 70% (close to the normal 68%) of the data is within one standard deviation of the mean, and 96% (close to the normal 95%) of the data is within 2 standard deviations of the mean.

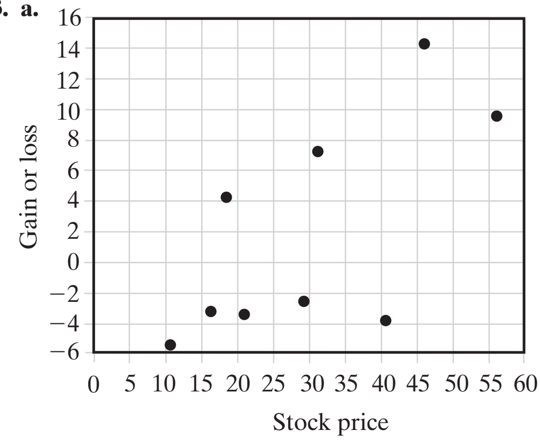
9. a. Moderate negative linear correlation
b. Negative

10. a. Strong positive linear correlation
b. Positive

11. a. Strong negative linear correlation
b. Negative

12. a. Moderate positive linear correlation
b. Positive

13. a.



b. Yes, moderate positive linear correlation

14. a. $y = 1.020x + 0.024$ **b.** $r = 0.999$

15. a. $y = 4x + 47.5$ **b.** $r = 0.970$

16. $y = 102.722(1.166)^x$

17. $y = 699.397 - 250.239 \ln x$

18. 24.4 million people **19.** $67,500

20. 351 dozen cookies

21. a. 179 deer **b.** In the 7th year

Exploration

1. $y = 13.619x^{2.122}$

2. $y = 35.938 + 1.627 \ln x$

Cumulative Review *(pages 669–671)*

Part I

1. 2 **2.** 3 **3.** 2
4. 1 **5.** 4 **6.** 4
7. 1 **8.** 2 **9.** 3
10. 4

Part II

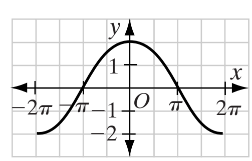
11. $x^2 - 6x + 13 = 0$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{6 \pm 4i}{2}$

$x = 3 \pm 2i$

12.



Part III

13. $\log \frac{\sqrt[3]{6}}{9} = \frac{1}{3}\log 6 - \log 9$

$= \frac{1}{3}(\log 2 + \log 3) - \log 3^2$

$= \frac{1}{3}(\log 2 + \log 3) - 2\log 3$

$= \frac{1}{3}(a + b) - 2b$

$= \frac{1}{3}a - \frac{5}{3}b$

14. $27^{x+1} = 81^x$

$(3^3)^{x+1} = (3^4)^x$

$3^{3x+3} = 3^{4x}$

$3x + 3 = 4x$

$x = 3$

Part IV

15. Let T = the top of the tree.
Let C = the base of the tree.

$m\angle BAT = 180 - 50 = 130°$

$m\angle ATB = 180 - 130 - 40 = 10°$

$\frac{20}{\sin 10°} = \frac{AT}{\sin 40°}$

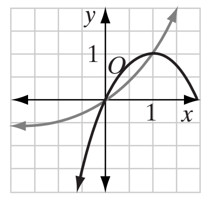
$AT = 74.033$

$\sin 50° = \frac{TC}{AT}$

$TC \approx 56.71$ ft

To the nearest foot, the height of the tree is 57 feet.

16.



Solutions: (0, 0) and (1, 1)

Chapter 16. Probability and the Binomial Theorem

16-1 The Counting Principle
(pages 675–678)

Writing About Mathematics

1. In the first situation, choosing a boy and choosing a girl are independent events. In the second situation, the choice of the first girl affects the choice of the second girl, and so the events are dependent.
2. The first is a dependent event, the second is an independent event. In the first situation, there are $52 \times 51 = 2{,}652$ possible outcomes. In the second, there are $52 \times 52 = 2{,}704$ possible outcomes.

Developing Skills

3. 24 **4.** 120 **5.** 336
6. 132 **7.** 256 **8.** 625
9. 64 **10.** 125 **11.** 16 events
12. Independent **13.** Dependent
14. Dependent **15.** Independent
16. Independent **17.** Dependent
18. 216 **19.** 32 **20.** 60
21. 48 **22.** 5,040 **23.** 30
24. 465 **25.** 3,993,600 **26.** 120
27. 12

Applying Skills

28. 6,720 **29.** 336
30. 15,120 **31.** 25
32. a. 462 **b.** 484
33. 756
34. a. 72 **b.** 18 **c.** 36
35. 256
36. a. 24 **b.** 12
37. a. 720 **b.** 120
38. a. 4,096 **b.** 4
39. a. 720 **b.** 240
40. a. 10,000 **b.** 5,040
c. Of the 10,000 telephone numbers with this prefix, 5,000 form an even number.
41. 79 **42.** 69

16-2 Permutations and Combinations
(pages 685–687)

Writing About Mathematics

1. ${}_nC_r = \frac{{}_nP_r}{r!} = \frac{\frac{n!}{(n-r)!}}{r!} = \frac{n!}{(n-r)! \times r!}$

2. $n! = n(n-1)(n-2)\cdots 1 = n(n-1)!$

Developing Skills

3. 120 **4.** 479,001,600 **5.** 6,720
6. 9 **7.** 604,800 **8.** 720
9. 1,680 **10.** 720 **11.** 20
12. 4 **13.** 792 **14.** 792
15. 210 **16.** 3,003 **17.** 3,003
18. 1 **19.** 1 **20.** 120
21. 1 **22.** 1 **23.** 720
24. 360 **25.** 120 **26.** 3,360
27. 40,320 **28.** 37,800 **29.** 50,400
30. 4,989,600

31. Order is not important and the chips are taken without replacement.

a. 84 **b.** 1 **c.** 8,190
d. 10,080 **e.** 1,170 **f.** 3,240
g. 336 **h.** 756

32. 5,040 **33.** 15,120 **34.** 13,860
35. 1,814,400 **36.** 4,200 **37.** 720
38. $\frac{1}{2}$ **39.** 10 **40.** 5
41. 10 **42.** 15 **43.** 12

Applying Skills

44. ${}_nC_{n-r} = \frac{n!}{[n-(n-r)]!(n-r)!} = \frac{n!}{r!(n-r)!} = {}_nC_r$

45. 56 **46.** 120
47. 18,876 **48.** 144
49. $\frac{10!}{4!} = 151,200$ **50.** 2,598,960
51. 116,396,280 **52.** 10
53. 3,024
54. **a.** 5,040 **b.** $\frac{1}{7}$ **c.** 1

55. **a.** $\frac{96!}{(90!)6!} = 927,048,304$

b. $\frac{96!}{(93!)3!} \cdot \frac{12!}{(3!)(9!)} = 31,433,600$

c. 924

56. **a.** 720 **b.** 120
57. 60,060

16-3 Probability *(pages 691–694)*

Writing About Mathematics

1. **a.** Yes. $\frac{1,954}{2,000} = .977 = 97.7\%$

b. Empirical, since it is based on real data.

2. Yes. Since the total probability of someone getting the part must equal 1, the probability of it being Casey is 0.4.

Developing Skills

3. $\frac{1}{6}$ **4.** $\frac{2}{3}$ **5.** $\frac{5}{36}$

6. **a.** 35,960 **b.** $\frac{1}{35,960} \approx .000028$

7. $\frac{1}{13}$ **8.** $\frac{1}{221}$

9. $\frac{1}{5,525}$ **10.** $\frac{2}{12} = \frac{1}{6}$

11. $\frac{12}{180} = \frac{1}{15}$ **12.** $\frac{\frac{9!}{2!2!}}{\frac{11!}{2!2!2!}} = \frac{1}{55}$

13. **a.** $\frac{1}{6}$ **b.** $\frac{1}{4}$ **c.** $\frac{2}{3}$ **d.** $\frac{1}{2}$

14. **a.** $\frac{3}{5}$ **b.** $\frac{1}{10}$ **c.** $\frac{19}{20}$ **d.** $\frac{7}{20}$

Applying Skills

15. .4

16. Heads $= \frac{526}{1,000} = .526$, tails $= \frac{474}{1,000} = .474$

17. **a.** $\frac{429}{1,200} = .3575$

b. No. The theoretical probability for rolling a 5 is $\frac{1}{6} = .1\overline{6}$. The die may be rigged.

18. 16%

19. **a.** Probability of *exactly* 2 plain:
$\frac{{}_{10}C_2 \times {}_{10}C_1}{{}_{20}C_3} = \frac{450}{1,140} \approx .3947$

b. Probability of *exactly* 1 maple, *exactly* 1 apple-cinnamon: $\frac{{}_6C_1 \times {}_4C_1 \times {}_{10}C_1}{{}_{20}C_3} = \frac{240}{1,140} \approx .2105$

c. $\frac{{}_{10}C_2 \times {}_6C_1}{{}_{20}C_3} = \frac{270}{1,140} \approx .2368$

d. $\frac{{}_{10}C_1 \times {}_6C_1 \times {}_4C_1}{{}_{20}C_3} = \frac{240}{1,140} \approx .2105$

20. 99.6%

21. **a.** $\frac{{}_3C_1 \times {}_{15}C_2}{{}_{18}C_3} = \frac{315}{816} \approx 0.3860$

b. There is 1 way to choose Stephanie.
There are 14 pairs involving Jan.
There are ${}_3C_1 \times {}_{15}C_2 = 315$ possible choices.
$\frac{1 \times 14}{315} = \frac{2}{45} = 0.\overline{4}$ *Answer*

22. $\frac{{}_6C_3}{{}_{18}C_3} = \frac{120}{4,896} \approx 0.0245$

23. **a.** $\frac{{}_{48}C_4}{{}_{50}C_4} = \frac{207}{245} \approx 0.8449$

b. $\frac{{}_{18}C_5}{{}_{50}C_5} = \frac{153}{37,835} \approx 0.0040$

24. **a.** $\frac{A_{bull}}{A_{board}} = \frac{\pi}{36^2} \approx 0.0024$

b. $\frac{72}{1,270} \approx 0.0567$

c. No, the empirical probability is much higher than the theoretical probability. This is likely because many players have some skill and therefore have a better than random chance of hitting the bull's-eye.

25. 16 in.^{2}

26. **a.** $\frac{5}{6} \cdot \frac{5}{6} \cdot \frac{1}{6} = \frac{5^2}{6^3} \approx .1157$

b. $\frac{5}{6} \cdot \frac{5}{6} \cdot \frac{5}{6} \cdot \frac{5}{6} \cdot \frac{1}{6} = \frac{5^4}{6^5} \approx .0804$

c. $\frac{5^{n-1}}{6^n}$

d. From the answer in part **c**, we can see that the common ratio is $\frac{5}{6}$.

27. **a.** $\frac{{}_{20}C_3}{{}_{24}C_3} = \frac{1,140}{2,024} = \frac{285}{506} \approx .56$

b. $\frac{{}_{20}C_2 \times {}_4C_1}{{}_{24}C_3} = \frac{760}{2,024} = \frac{95}{253} \approx .38$

28. **a.** $\frac{4}{16} = 0.25$

b. $\frac{4}{16} = 0.25$

16-4 Probability with Two Outcomes *(pages 699–700)*

Writing About Mathematics

1. $\frac{{}_{12}C_2 \times {}_8C_3}{{}_{20}C_5} = \frac{3,696}{15,504} \approx .2384$. This is not a Bernoulli experiment because each student is chosen without replacement and so the choices are not independent.
2. No, ${}_nC_r = \frac{n!}{r!(n-r)!} \neq \frac{n!}{r!}$.

Developing Skills

3. **a.** .15625 **b.** .3125 **c.** .3125 **d.** .15625 **e.** .03125 **f.** .03125 **g.** Two or three heads
4. **a.** 0.4019 **b.** 0.1608 **c.** 0.0322 **d.** 0.0032 **e.** 0.0001 **f.** 0.4019 **g.** One or zero sixes
5. **a.** 0.2420 **b.** 0.0302 **c.** 0.0017 **d.** 0.00004 **e.** 0.7260 **f.** No kings
6. **a.** 0.0584 **b.** 0.1877

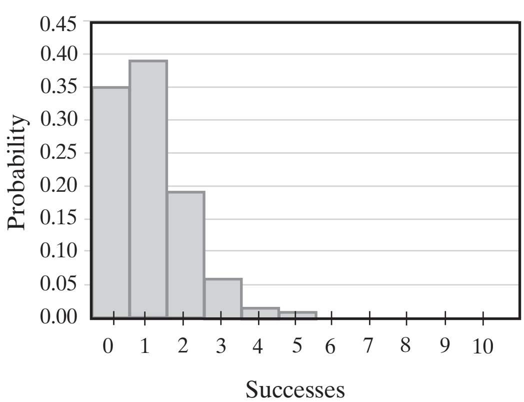
Applying Skills

7. ${}_7C_5\left(\frac{2}{3}\right)^5\left(\frac{1}{3}\right)^2 \approx .3073$
8. ${}_3C_1\,(.2)^1(.8)^2 \approx .3840$
9. ${}_4C_3\,(.95)^3(.05)^1 \approx .1715$
10. **a.** .2 **b.** ${}_3C_1\,(.2)^1(.8)^2 = .384$
11. ${}_5C_2\,(.04)^2(.96)^3 = .0142$
12. ${}_{20}C_2\left(\frac{1}{12}\right)^2\left(\frac{11}{12}\right)^{18} = .2755$
13. ${}_5C_5\,(.92)^5\,(.08)^2 = .6591$
14. ${}_4C_4\,(.65)^4\,(.35)^0 = .1785$

16-5 Binomial Probability and the Normal Curve *(pages 706–708)*

Writing About Mathematics

1. No. Exactly r is included in both "at least" and "at most," so their sum will be greater than 1.
2. No, as you can see from the histogram of the probabilities, the graph is not bell-shaped.



Developing Skills

3. **a.** $\frac{3}{16}$ **b.** $\frac{1}{2}$ **c.** $\frac{13}{16}$ **d.** $\frac{31}{32}$
4. **a.** $\frac{25}{27}$ **b.** $\frac{215}{216}$ **c.** $\frac{2}{27}$ **d.** $\frac{91}{216}$
5. **a.** $\frac{3}{5}$ **b.** $\frac{297}{625}$ **c.** $\frac{513}{625}$ **d.** $\frac{609}{625}$
6. $\sum_{r=10}^{15} {}_{15}C_r\left(\frac{1}{2}\right)^r\left(\frac{1}{2}\right)^{15-r} = \sum_{r=10}^{15} {}_{15}C_r\left(\frac{1}{2}\right)^{15}$
7. $\sum_{r=0}^{7} {}_{10}C_r\left(\frac{1}{2}\right)^r\left(\frac{1}{2}\right)^{10-r} = \sum_{r=0}^{7} {}_{10}C_r\left(\frac{1}{2}\right)^{10}$
8. $\sum_{r=5}^{20} {}_{20}C_r\left(\frac{2}{3}\right)^r\left(\frac{1}{3}\right)^{20-r}$
9. $\sum_{r=0}^{3} {}_{10}C_r\left(\frac{1}{3}\right)^r\left(\frac{2}{3}\right)^{10-r}$
10. .93 **11.** .49 **12.** .54
13. .84; for the upper limit, use any value more than 3 standard deviations above the mean.
14. .224 **15.** .453
16. .176 **17.** .045

Applying Skills

18. $\frac{1}{2}$ **19.** .9998 **20.** .4315
21. **a.** .0460 **b.** .1056
22. .382 **23.** .141 **24.** .655

16-6 The Binomial Theorem *(pages 710–711)*

Writing About Mathematics

1. ${}_nC_r = {}_nC_{n-r}$
2. Yes, $\left(x + \frac{1}{x}\right)^n = \sum_{i=0}^{n} {}_nC_i\,x^{n-i}\left(\frac{1}{x}\right)^i$
$= \sum_{i=0}^{n} {}_nC_i\,x^{n-i}x^{-i} \sum_{i=0}^{n} {}_nC_i\,x^{n-2i}$.

Developing Skills

3. $x^6 + 6x^5y + 15x^4y^2 + 20x^3y^3 + 15x^2y^4 + 6xy^5 + y^6$
4. $x^7 + 7x^6y + 21x^5y^2 + 35x^4y^3 + 35x^3y^4 + 21x^2y^5 + 7xy^6 + y^7$
5. $1 + 5y + 10y^2 + 10y^3 + 5y^4 + y^5$
6. $x^5 + 10x^4 + 40x^3 + 80x^2 + 80x + 32$
7. $a^4 + 12a^3 + 54a^2 + 108a + 81$
8. $16 + 32a + 24a^2 + 8a^3 + a^4$
9. $8b^3 - 12b^2 + 6b - 1$
10. $-4 + 4i$

11.

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1
1 8 29 56 70 56 29 8 1
1 9 37 85 126 126 85 37 9 1

12. ${}_{15}C_2x^{13}y^2 = 105x^{13}y^2$
13. ${}_{10}C_6x^4y^6 = 210x^4y^6$
14. ${}_6C_3(2x)^3(y)^3 = 160x^3y^3$
15. ${}_9C_4(x)^5(-y)^4 = 126x^5y^4$
16. ${}_7C_5(3a)^2(2b)^5 = 6{,}048a^2b^5$
17. ${}_8C_3(y)^5\left(-\frac{1}{y}\right)^3 = -56y^2$
18. 6th term, 7th term $= 1{,}792a^2b^6$
19. 4th term, 5th term $= 16d^4$
20. 5th term, 6th term $= -231x^6y^5$

Applying Skills

21. $27x^3 - 27x^2 + 9x - 1$

22. $100\sum_{i=0}^{12} {}_{12}C_i(1)^{12-i}(.01)^i = 100\sum_{i=0}^{12} {}_{12}C_i(.01)^i$

23. $75{,}000\sum_{i=0}^{5} {}_5C_i(1)^{5-i}(-.20)^i = 75{,}000\sum_{i=0}^{5} {}_5C_i(-.20)^i$

Review Exercises *(pages 713–714)*

1. 720 **2.** 120 **3.** 1
4. 20,160 **5.** 28 **6.** 1
7. 1 **8.** 210 **9.** 50
10. 132 **11.** 792 **12.** 40
13. 40,320 **14.** 19,958,400 **15.** 360
16. 120 **17.** 56
18. **a.** .2036 **b.** .0011 **c.** .7738 **d.** .1887
19. **a.** $\frac{33}{2{,}048} \approx .0161$ **b.** $\frac{79}{4{,}096} \approx .0193$ **c.** $\frac{4{,}083}{4{,}096} \approx .9968$
20. .868
21. **a.** .021 **b.** .023 **c.** .683
22. $x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4$
23. $128a^7 + 448a^6 + 672a^5 + 560a^4 + 280a^3 + 84a^2 + 14a + 1$
24. $729 - 1{,}458x + 1{,}215x^2 - 540x^3 + 135x^4 - 18x^5 + x^6$
25. $b^8 - 8b^6 + 28b^4 - 56b^2 + 70 - \frac{56}{b^2} + \frac{28}{b^4} - \frac{8}{b^6} + \frac{1}{b^8}$
26. $-4 - 4i$
27. ${}_9C_3x^6(-y)^3 = -84x^6y^3$
28. ${}_{10}C_6a^4(3)^6 = 153{,}090a^4$
29. ${}_{12}C_6(2x)^6(-1)^6 = 59{,}136x^6$

Exploration *(pages 714–715)*

a. This calculation assumes that 7 games will be played and is the probability that the American League team will win 4 games. The World Series is won by the first team to win 4 games, and so as few as 4 games may be played.
b. ${}_4C_4p^4 = p^4$
c. ${}_4C_1p^4(1-p) = 4p^4(1-p)$
d. ${}_5C_2p^4(1-p)^2 = 10p^4(1-p)^2$
e. ${}_6C_3p^4(1-p)^3 = 20p^4(1-p)^3$
f. $p^4 + 4p^4(1-p) + 10p^4(1-p)^2 + 20p^4(1-p)^3$

Cumulative Review *(pages 715–717)*

Part I

1. 4 **2.** 3 **3.** 4
4. 1 **5.** 4 **6.** 1
7. 4 **8.** 1 **9.** 3
10. 2

Part II

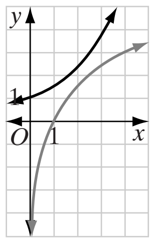
11.
$$\log_2 \tfrac{1}{4} = \log_8 x$$
$$-2 = \log_8 x$$
$$x = 8^{-2} = \tfrac{1}{64}$$

12. *Answer:* $x = 1$
$$\sqrt{3x + 1} = 2x$$
$$3x + 1 = 4x^2$$
$$0 = 4x^2 - 3x - 1$$
$$0 = (x - 1)(4x + 1)$$
$$x = 1, -\tfrac{1}{4}$$

Check $x = 1$	*Check* $x = -\frac{1}{4}$
$\sqrt{3(1) + 1} \stackrel{?}{=} 2(1)$	$\sqrt{3\left(-\frac{1}{4}\right) + 1} \stackrel{?}{=} 2\left(-\frac{1}{4}\right)$
$\sqrt{4} \stackrel{?}{=} 2$	$\sqrt{\frac{1}{4}} \stackrel{?}{=} -\frac{1}{2}$
$2 = 2$ ✔	$\frac{1}{2} \neq -\frac{1}{2}$ ✘

Part III

13. a–b.



$f^{-1}(x) = \frac{\ln x}{\ln(1.6)} = \log_{1.6} x$

14. *Answer:* $-\frac{5}{2} < w < 4$

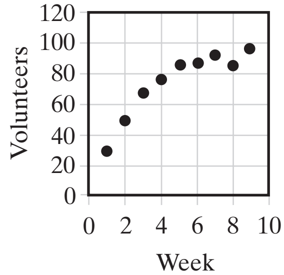
$$l = 2w - 3$$
$$A = lw = 2w^2 - 3w < 20$$
$$2w^2 - 3w - 20 < 0$$
$$(2w + 5)(w - 4) < 0$$

The solutions to the corresponding equality are $-\frac{5}{2}$ and 4.

The original inequality is true in the interval $-\frac{5}{2} < w < 4$.

Part IV

15. a.



b. $y = 31.1327 + 31.1523 \ln x$ **c.** 103

16.

$$2\cos^2\theta + 2\sin\theta - 1 = 0$$
$$2(1 - \sin^2\theta) + 2\sin\theta - 1 = 0$$
$$-2\sin^2\theta + 2\sin\theta + 1 = 0$$
$$\sin\theta = \frac{-2 \pm \sqrt{4 + 8}}{-4}$$
$$\sin\theta = \frac{1}{2} \pm \frac{\sqrt{3}}{2}$$
$$\theta = \arcsin\left(\frac{1 - \sqrt{3}}{2}\right)$$
$$\theta = 201°, 339°$$