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Name: Key **Unit 4 Review – Rationals**

Helpful Information:

Multiplying – Factor and cancel common factors between the numerator and denominator

Dividing – Keep Change Flip (aka multiply by the reciprocal)

Adding/Subtracting – Find common denominators

Synthetic Division (uses the root) is a shortcut for **Long Division (uses the whole polynomial)** that can *only* be used when the leading coefficient of the binomial you divide by is 1

If something is a **factor** of a polynomial you can check by (remember if it is short answer you need to be able to show work):

- ✓ Synthetic division will have no remainder
- ✓ Long Division will have no remainder
- ✓ Evaluating the function at the root will give you 0
- ✓ Look for a root on the graph (where it crosses the x-axis)
- ✓ Look in the table for the x-value when y=0

Rational Root Theorem - potential roots of a polynomial are found by finding all the positive and negative values of the factors of the last term divided by the factors of the first term

Level I Practice:

1. The zeros for $f(x) = x^4 - 4x^3 - 9x^2 + 36x$ are

could graph in Calculator

1) $\{0, \pm 3, 4\}$
2) $\{0, 3, 4\}$
3) $\{0, \pm 3, -4\}$
4) $\{0, 3, -4\}$

$x(x^3 - 4x^2 / - 9x + 36)$

$x[x^2(x-4) - 9(x-4)]$

$x(x^2-9)(x-4)$

$x(x-3)(x+3)(x-4)$

$x=0 \quad x=3 \quad x=-3 \quad x=4$

2. Solve for x: $\frac{1}{x} - \frac{1}{3} = -\frac{1}{3x}$

LCD $3x$

$\frac{(3)1}{(3)x} - \frac{1(x)}{3(x)} \quad -\frac{1}{3x}$

$\frac{3}{3x} - \frac{x}{3x} = \frac{-1}{3x}$

$\frac{3-x}{3x} = \frac{-1}{3x}$

common denominata
drop denominata

$3 - x = -1$
$+x \qquad +x$
$3 = -1 + x$
$+1 \qquad +1$
$4 = x$

ans. $x=4$

must Check:

$\frac{1}{x} - \frac{1}{3} = -\frac{1}{3x}$

$\frac{1}{4} - \frac{1}{3} = \frac{-1}{12}$

$\frac{3}{12} - \frac{4}{12} = -\frac{1}{12}$ ✓

3. When $g(x)$ is divided by $x+4$, the remainder is 0. Given $g(x) = x^4 + 3x^3 - 6x^2 - 6x + 8$, which conclusion about $g(x)$ is true?

1) $g(4) = 0$
2) $g(-4) = 0$
3) $x - 4$ is a factor of $g(x)$.
4) No conclusion can be made regarding $g(x)$.

$x+4$ is a facta
$\therefore -4$ is a root
$\therefore g(-4) = 0$

Level II Practice:

4. The expression $\frac{4x^3 + 5x + 10}{2x + 3}$ is equivalent to

only use long ÷ because of the $2x$

1) $2x^2 + 3x - 7 + \frac{31}{2x+3}$
2) $2x^2 - 3x + 7 - \frac{11}{2x+3}$
3) $2x^2 + 2.5x + 5 + \frac{15}{2x+3}$
4) $2x^2 - 2.5x - 5 - \frac{20}{2x+3}$

$$
\begin{array}{r}
2x^2 - 3x + 7 \\
2x+3 \overline{)\, 4x^3 + 0x^2 + 5x + 10} \\
-\; 4x^3 \oplus 6x^2 \qquad\qquad \\
\hline
-6x^2 + 5x \qquad \\
+ \ominus 6x^2 \ominus 9x \qquad \\
\hline
14x + 10 \\
- \; 14x \oplus 21 \\
\hline
-11
\end{array}
$$

5. Determine if $x - 5$ is a factor of $2x^3 - 4x^2 - 7x - 10$. Explain your answer.

$x-5$ is a factu
$x=5$, 5 is a root
$2(5)^3 - 4(5)^2 - 7(5) - 10 = 0$
$250 - 100 - 35 - 10 = 0$
$105 \neq 0$

or

$x-5=0$
$x=5$

$$
\begin{array}{r|rrr|r}
5 & 2 & -4 & -7 & -10 \\
 & \downarrow & 10 & 30 & 115 \\
\hline
 & 2 & 6 & 23 & 105
\end{array}
$$

when ÷ 105 is a remainder. A remainder show the divise is not a root

if $(x-5)$ is a factor $x=5$ is a root & when substituted into eq would $=0$.

6. Algebraically prove that $\frac{x^3+9}{x^3+8} = 1 + \frac{1}{x^3+8}$, where $x \neq -2$.

Show the right = the left.

$$\frac{x^3+9}{x^3+8} = 1 + \frac{1}{x^3+8}$$

$$\frac{x^3+9}{x^3+8} = \frac{x^3+8}{x^3+8} + \frac{1}{x^3+8}$$

$$\frac{x^3+9}{x^3+8} = \frac{x^3+8+1}{x^3+8}$$

$$\frac{x^3+9}{x^3+8} = \frac{x^3+9}{x^3+8}$$

Level III Practice:

7. What is the solution, if any, of the equation $\frac{2}{x+3} - \frac{3}{4-x} = \frac{2x-2}{x^2-x-12}$?

$(x-4)(x+3)$

1) -1
2) -5
3) all real numbers
4) no real solution

$\frac{2(x-4)}{(x+3)(x-4)} - \frac{3(x+3)}{-1(x-4)(x+3)} = \frac{2x-2}{(x-4)(x+3)}$

$\frac{2(x-4)}{(x+3)(x-4)} + \frac{3(x+3)}{(x-4)(x+3)} = \frac{2x-2}{(x-4)(x+3)}$

$2(x-4) + 3(x+3) = 2x-2$

$2x - 8 + 3x + 9 = 2x - 2$

$5x + 1 = 2x - 2$

$-2x \qquad -2x$

$3x+1 = -2$

$-1 \qquad -1$

$3x = -3$

$\boxed{x=-1}$

Check ans.

Check w/ $x=-1$

$\frac{2}{-1+3} - \frac{3}{4+1} = \frac{2(-1)-2}{(-1)^2+1-12}$

$\frac{2}{2} - \frac{3}{5} = \frac{-4}{-10} = \frac{2}{5}$

$1 - \frac{3}{5} = \frac{2}{5}$

$\frac{2}{5} = \frac{2}{5}$ ✓

8. Given $f(x) = 3x^2 + 7x - 20$ and $g(x) = x - 2$, state the quotient and remainder of $\frac{f(x)}{g(x)}$, in the form $q(x) + \frac{r(x)}{g(x)}$. J quotient & remainder

$\boxed{3x + 13 + \frac{6}{x-2}}$

$$\begin{array}{r}
3x + 13 \\
x-2 \,\big)\, 3x^2 + 7x - 20 \\
-\,3x^2 - 6x \quad \\
\hline
13x - 20 \\
-13x - 26 \\
\hline
6
\end{array}$$

OK

$$\begin{array}{r|rr|r}
2 & 3 & 7 & -20 \\
 & \downarrow & +6 & 26 \\
\hline
 & 3 & 13 & 6
\end{array}$$

$\boxed{3x + 13 + \frac{6}{x-2}}$