

Name: Key

Unit 6 Review – Trigonometry & the Unit Circle

Helpful Information:

	$\sin \theta$	$\cos \theta$	$\tan \theta$
$0^\circ / 0 \text{ rad}$	$\frac{\sqrt{0}}{2} = 0$	$\frac{\sqrt{4}}{2} = 1$	0
$30^\circ / \frac{\pi}{6} \text{ rad}$	$\frac{\sqrt{1}}{2} = \frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$
$45^\circ / \frac{\pi}{4} \text{ rad}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1
$60^\circ / \frac{\pi}{3} \text{ rad}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{1}}{2} = \frac{1}{2}$	$\sqrt{3}$
$90^\circ / \frac{\pi}{2} \text{ rad}$	$\frac{\sqrt{4}}{2} = 1$	$\frac{\sqrt{0}}{2} = 0$	und.

Reference angles always drawn from the x-axis

$(x, y) \rightarrow (\cos \theta, \sin \theta)$

Trig Identities & Pythagorean Identities

- $\sin^2 x + \cos^2 x = 1$
- $\tan x = \frac{\sin x}{\cos x}$
- $\csc x = \frac{1}{\sin x}$
- $\sec x = \frac{1}{\cos x}$
- $\cot x = \frac{1}{\tan x} = \frac{\cos x}{\sin x}$

$$1 \text{ radian} = 180^\circ$$



Remember what is positive in each quadrant:

- All Students Take Calculus
- Alvin, Simon, and Theodore are the Chipmunks

Level I Practice:

1. If $\sin^2(32^\circ) + \cos^2(M) = 1$, then M equals

1) 32°

2) 58°

3) 68°

4) 72°

2. Tarisha and Alan are writing an expression for the measure of an angle coterminal with the angle shown at the right. Is either of them correct? Explain your reasoning.

Tarisha	Alan
The measure of a coterminal angle is $(x - 360)^\circ$	The measure of a coterminal angle is $(360 - x)^\circ$

Both correct

Tarisha is referencing an angle $> 360^\circ$

Alan is referencing a negative angle (going in clockwise direction)

3. Determine whether $3 \sin 60^\circ = \sin 180^\circ$ is true or false. Explain your reasoning.

False: 3 times (value of $\sin 60^\circ$)

$$3\left(\frac{\sqrt{3}}{2}\right) \neq 0$$

$$\frac{3\sqrt{3}}{2} \neq 0$$

Or

$$3(.8660) \neq 0$$

$$2.5980 \neq 0$$

4. In the diagram below of right triangle KTW , $KW=6$, $KT=5$, and $m\angle KTW=90$. What is the measure of $\angle K$, to the nearest minute?

$$\cos K = \frac{5}{6}$$

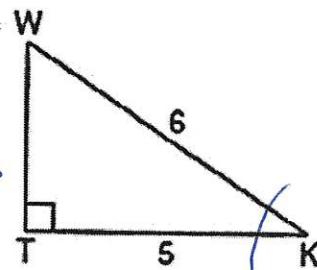
$$K = \cos^{-1}\left(\frac{5}{6}\right)$$

$$K = 33.55730976$$

2nd Angle \rightarrow 4: DMS

$$23^\circ 33' 26.315'$$

$$23^\circ 33'$$



a. $33^\circ 33'$

b. $33^\circ 34'$

c. $33^\circ 55'$

d. $33^\circ 56'$

Level II Practice:

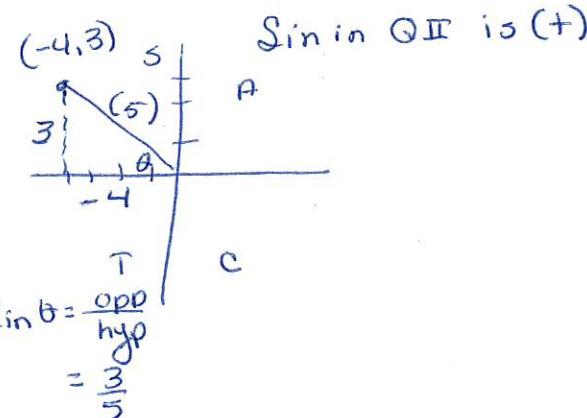
5. If the terminal side of angle θ , in standard position, passes through point $(-4, 3)$, what is the numerical value of $\sin \theta$?

1) $\frac{3}{5}$

2) $\frac{4}{5}$

3) $-\frac{3}{5}$

4) $-\frac{4}{5}$



6. A shadow moves around a sundial 15° every hour.

- a. After how many hours is the angle of rotation of the shadow $\frac{8\pi}{5}$ radians?

Radian \rightarrow degrees

$$\frac{8\pi}{5} \cdot \frac{180}{\pi} = 288^\circ$$

$$\frac{288^\circ}{15^\circ} = 19.2 \text{ hrs}$$

- b. What is the angle of rotation in radians after 5 hours?

$$15^\circ \times 5 = 75^\circ \text{ in hrs.}$$

degrees \rightarrow Radians

$$75^\circ \cdot \frac{\pi}{180} = \frac{75\pi}{180} = \frac{5\pi}{12} \text{ radians}$$

7. Jordan and Ebony are simplifying $\frac{\sin^2\theta}{\cos^2\theta + \sin^2\theta}$. Is either correct? Explain your reasoning.

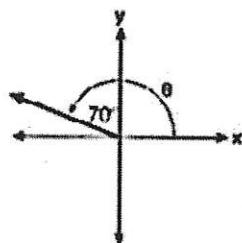
Can't split a denominator like this!

Jordan	Ebony
$\frac{\sin^2\theta}{\cos^2\theta + \sin^2\theta}$ $= \frac{\sin^2\theta}{\cos^2\theta} + \frac{\sin^2\theta}{\sin^2\theta}$ $= \tan^2\theta + 1$ $= \sec^2\theta$	$\frac{\sin^2\theta}{\cos^2\theta + \sin^2\theta}$ $= \frac{\sin^2\theta}{1}$ $= \sin^2\theta$

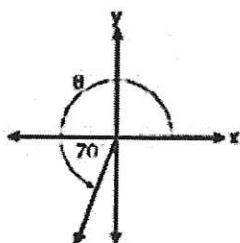
Ebony is correct.

8. In which graph is θ coterminal with an angle of -70° ?

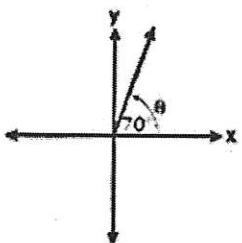
(1)



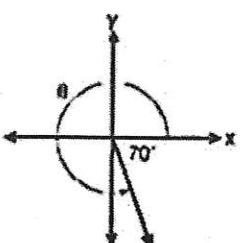
(3)



(2)

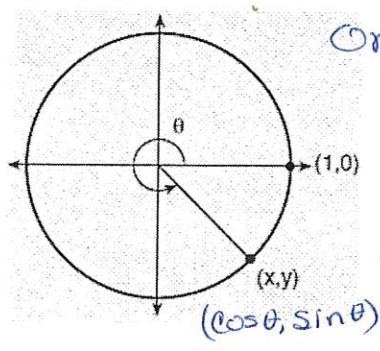


(4)



Level III Practice:

9. Using the unit circle below, explain why $\csc\theta = \frac{1}{y}$.



On the unit circle, each point (x, y) is actually $(\cos\theta, \sin\theta)$.

$$x = \cos\theta \quad y = \sin\theta$$

$$\text{Since } \csc\theta = \frac{1}{\sin\theta}$$

$$\csc\theta = \frac{1}{y}$$

10. Simplify the following expression by writing it in terms of $\sin \theta$:

$$\frac{\sec \theta}{\cot \theta + \tan \theta}$$

$$\begin{aligned} & \sec \theta \div (\cot \theta + \tan \theta) \\ & \frac{1}{\cos \theta} \div \left(\frac{\cos \theta (\cot \theta)}{\sin \theta (\cot \theta)} + \frac{\sin \theta (\tan \theta)}{\cos \theta (\tan \theta)} \right) \\ & \frac{1}{\cos \theta} \div \left(\frac{\cos^2 \theta + \sin^2 \theta}{\sin \theta \cos \theta} \right) \\ & \frac{1}{\cos \theta} \cdot \frac{\sin \theta \cos \theta}{\cos^2 \theta + \sin^2 \theta} \\ & \frac{1}{\cos \theta} \cdot \frac{\sin \theta \cancel{\cos \theta}}{1} = \underline{\sin \theta} \end{aligned}$$

11. A circle centered at the origin has a radius of 10 units. The terminal side of an angle, θ , intercepts the circle in Quadrant II at point C. The y-coordinate of point C is 8. What is the value of $\cos \theta$?

1) $-\frac{3}{5}$

2) $-\frac{3}{4}$

3) $\frac{3}{5}$

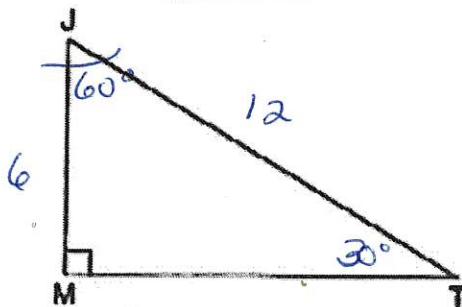
4) $\frac{4}{5}$

$$\begin{aligned} a^2 + b^2 &= c^2 \\ x^2 + y^2 &= c^2 \\ x^2 + 8^2 &= 10^2 \\ x^2 + 64 &= 100 \\ x^2 &= 36 \\ x &= 6 \end{aligned}$$

$$\cos \theta = \frac{\text{adj}}{\text{hyp}} + \text{neg in QII}$$

$$\cos \theta = \frac{-6}{10} = -\frac{3}{5}$$

12. In the diagram below of right triangle JTM , $\underline{JT}=12$, $\underline{JM}=6$, and $m\angle JMT=90$. What is the value of $\underline{\cot J}$?



value of $\cot J$?

a. $\frac{\sqrt{3}}{3}$

b. 2

c. $\sqrt{3}$

d. $\frac{2\sqrt{3}}{3}$

$$\cos J = \frac{6}{12} = \frac{1}{2}$$

$$m\angle J = 60^\circ$$

$$\sin 60^\circ = \frac{\sqrt{3}}{2}$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta} \quad \text{So} \quad \cot J = \frac{\cos \theta}{\sin \theta}$$

$$= \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}}$$

$$= \frac{1}{2} \div \frac{\sqrt{3}}{2}$$

$$= \frac{1}{2} \cdot \frac{2}{\sqrt{3}}$$

$$= \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$